#1: Studio della metrica di Schwarzschild (in 2 dimensioni: dt=0, dz=0)

#2: CaseMode := Sensitive

#3: InputMode := Word

#4: Equazioni parametriche della superficie, R:=R(r,θ)

#5: $R := [r \cdot \mathrm{COS}(\theta),\ r \cdot \mathrm{SIN}(\theta),\ 2 \cdot \sqrt{(2 \cdot GMC2)} \cdot \sqrt{(r - 2 \cdot GMC2)}]$

#6: $r :\in \mathrm{Real}\ [0, \infty)$

#7: $\theta :\in \mathrm{Real}\ [-\pi, \pi]$

#8: $GMC2 := G\ M / c^2$

#9: $GMC2 :\in \mathrm{Real}\ (0, \infty)$

#10: Calcolo delle derivate prime

#11: $$R\_r := \frac{d}{dr} R$$

#12: $$R\_r := \left[\mathrm{COS}(\theta),\ \mathrm{SIN}(\theta),\ \frac{\sqrt{2} \cdot \sqrt{GMC2}}{\sqrt{(r - 2 \cdot GMC2)}}\right]$$

#13: $$R\_\theta := \frac{d}{d\theta} R$$

#14: $R\_\theta := [-r \cdot \mathrm{SIN}(\theta),\ r \cdot \mathrm{COS}(\theta),\ 0]$

#15: Coefficienti della 1ª forma differenziale di Gauss

#16: $$E := R\_r^2$$

#17: $$E := \frac{1}{1 - \frac{2 \cdot GMC2}{r}}$$

#18: $F := R\_r \cdot R\_\theta$

#19: $F := 0$

#20: $$G := R\_\theta^2$$

#21: $$G := r^2$$

#22: 1ª forma differenziale di Gauss

$$\#23:\quad ds^2 = E\cdot dr^2 + 2\cdot F\cdot dr\cdot d\theta + G\cdot d\theta^2$$

$$\#24:\quad ds^2 = \frac{1}{1-\frac{2\cdot GMC2}{r}}\cdot dr^2 + d\theta^2\cdot r^2$$

#25: Calcolo delle derivate seconde

$$\#26:\quad R\_rr := \frac{d}{dr} R\_r$$

$$\#27:\quad R\_rr := \left[0,\ 0,\ -\frac{\sqrt{2}\cdot\sqrt{GMC2}}{2\cdot(r-2\cdot GMC2)^{3/2}}\right]$$

$$\#28:\quad R\_r\theta := \frac{d}{d\theta} R\_r$$

#29: R_rθ := [- SIN(θ), COS(θ), 0]

$$\#30:\quad R\_\theta r := \frac{d}{dr} R\_\theta$$

#31: R_θr := [- SIN(θ), COS(θ), 0]

$$\#32:\quad R\_\theta\theta := \frac{d}{d\theta} R\_\theta$$

#33: R_θθ := [- r·COS(θ), - r·SIN(θ), 0]

#34: Vettore normale

#35: VettNorm := CROSS(R_r, R_θ)

$$\#36:\quad VettNorm := \left[-\frac{\sqrt{2}\cdot\sqrt{GMC2}\cdot r\cdot COS(\theta)}{\sqrt{r-2\cdot GMC2}},\ -\frac{\sqrt{2}\cdot\sqrt{GMC2}\cdot r\cdot SIN(\theta)}{\sqrt{r-2\cdot GMC2}},\ r\right]$$

#37: Elemento di superficie

$$\#38:\quad dS := \sqrt{(E\cdot G - F^2)}\cdot dr\cdot d\theta$$

$$\#39:\quad dS := \frac{1}{\sqrt{\left(1-\frac{2\cdot GMC2}{r}\right)}}\cdot r\cdot dr\cdot d\theta$$

#40: Versore normale

$$\#41:\quad \text{VersNorm} := \frac{\text{CROSS}(R\_r,\ R\_\theta)}{\sqrt{(E\cdot G - F^2)}}$$

$$\#42:\quad \text{VersNorm} := \left[-\sqrt{\frac{2\cdot GMC2}{r}}\cdot\cos(\theta),\ -\sqrt{\frac{2\cdot GMC2}{r}}\cdot\sin(\theta),\ \sqrt{1-\frac{2\cdot GMC2}{r}}\right]$$

#43: Coefficienti della 2ª forma differenziale di Gauss

#44: $L := R\_rr\cdot\text{VersNorm}$

$$\#45:\quad L := \frac{\sqrt{2}\cdot\sqrt{GMC2}}{2\cdot\sqrt{r}\cdot(2\cdot GMC2 - r)}$$

#46: $M := R\_r\theta\cdot\text{VersNorm}$

#47: $M := 0$

#48: $N := R\_\theta\theta\cdot\text{VersNorm}$

#49: $N := \sqrt{2}\cdot\sqrt{GMC2}\cdot\sqrt{r}$

#50: Calcolo dei raggi di curvatura principali

$$\#51:\quad (E\cdot G - F^2)\cdot rc^{-2} + (2\cdot F\cdot M - E\cdot N - G\cdot L)\cdot rc^{-1} + (L\cdot N - M^2) = 0$$

$$\#52:\quad \sqrt{2}\cdot GMC2\cdot rc^2 + \sqrt{GMC2}\cdot rc\cdot r^{3/2} - \sqrt{2}\cdot r^3 = 0$$

$$\#53:\quad rc = -\frac{\sqrt{2}\cdot r^{3/2}}{\sqrt{GMC2}}$$

$$\#54:\quad rc = \frac{\sqrt{2}\cdot r^{3/2}}{2\cdot\sqrt{GMC2}}$$

#55: Curvatura di Gauss

$$\#56:\quad K := \frac{L\cdot N - M^2}{E\cdot G - F^2}$$

$$\#57:\quad K := -\frac{GMC2}{r^3}$$

#58: Curvatura media

$$\#59:\quad H := -\frac{2\cdot F\cdot M - E\cdot N - G\cdot L}{2\cdot(E\cdot G - F^2)}$$

$$\#60:\quad H := \frac{\sqrt{2}\cdot\sqrt{GMC2}}{4\cdot r^{3/2}}$$

#61: Direzioni dei versi normali

$$\#62:\quad (F\cdot N - G\cdot M)\cdot SIN(\alpha)^2 + (E\cdot N - G\cdot L)\cdot SIN(\alpha)\cdot COS(\alpha) + (E\cdot M - F\cdot L)\cdot COS(\alpha)^2 = 0$$

$$\#63:\quad \alpha = 0$$

$$\#64:\quad \alpha = \frac{\pi}{2}$$

$$\#65:\quad \alpha = \pi$$

$$\#66:\quad \alpha = \frac{3\cdot\pi}{2}$$