Applicando i teoremi sui logaritmi, trasformare, se è possibile, le seguenti espressioni in somme algebriche, qualunque sia la base:

72. $\log 6xyz$; $\quad \log 3a(c+d)$; $\quad \log(a+b)(m+n)$; $\quad \log(a^2-b^2)$; $\quad \log a(x^2-1)$; $\quad \log \dfrac{1}{ab}$.

73. $\log \dfrac{x^2-y^2}{2xy}$; $\quad \log \dfrac{3(a^2-b^2)x}{x+y}$; $\quad \log ab^2$; $\quad \log(ab)^2$; $\quad \log \dfrac{5mx}{1-m^2}$.

74. $\log 2a^3$; $\quad \log 5a^2x^3$; $\quad \log(a+b)^3$; $\quad \log \dfrac{ab-cd}{mn-pq}$; $\quad \log \dfrac{2a^8}{2bx^2}$.

75. $\log \dfrac{1}{2}$; $\quad \log \dfrac{1}{3}$; $\quad \log \dfrac{1}{7}$; $\quad \log 0{,}1$; $\quad \log \dfrac{2}{3}$; $\quad \log \dfrac{2}{7}$; $\quad \log \dfrac{7}{3}$.

76. $\log\left(\dfrac{2}{3\cdot 7}\right)^5$; $\quad \log\left[\left(\dfrac{3}{11}\right)^3 : \left(\dfrac{2}{7}\right)^4\right]$; $\quad \log 5x^2y$; $\quad \log \dfrac{3y^2}{2x}$.

77. $\log \dfrac{ab^3}{c^2}$; $\quad \log\left(\dfrac{a^2}{b}\right)^3$; $\quad \log\sqrt{3xy}$; $\quad \log\sqrt[3]{\dfrac{ab}{c^2}}$; $\quad \log \dfrac{1}{3a^2b^2c^4}$.

78. $\log \dfrac{1}{\sqrt[4]{a^3bcd^5}}$; $\quad \log\left(\dfrac{4}{3}\pi r^3\right)$; $\quad \log\left(2\pi\sqrt{\dfrac{1}{g}}\right)$; $\quad \log \dfrac{a^3b\sqrt[4]{c}}{d^3e^4\sqrt{l^3}}$; $\quad \log \dfrac{2a^3b^4c^{\frac{1}{2}}}{\sqrt[3]{4a^2b^4c^5}}$

79. $\log \dfrac{x^3-y^3}{2xy}$; $\quad \log \dfrac{a^3+b^3}{a^3-b^3}$; $\quad \log \sqrt[3]{a^2}\cdot\dfrac{\sqrt[3]{b}\cdot c}{a^2\cdot b}$; $\quad \log(\sqrt{a}\cdot\sqrt{a}\cdot\sqrt[3]{b})$.

80. $\log\sqrt{p(p-a)(p-b)(p-c)}$; $\quad \log\sqrt{\dfrac{(x+y)(y+z)}{x+z}}$; $\quad \log\sqrt[3]{\dfrac{xy^2z^3}{\sqrt{ab}}}$; $\quad \log \dfrac{x^3\sqrt{x^3y\sqrt[3]{xy^2}}}{\sqrt[3]{xy^2}}$.

81. $\log \dfrac{x^2y^3\sqrt{z^3\sqrt[3]{t}}}{a\sqrt{b}\cdot c\sqrt{d}}$; $\quad \log_x \dfrac{\sqrt[3]{x}}{\sqrt[7]{a^4-b^4}}$; $\quad \log(\sqrt[m]{x^n}\,\sqrt[p]{y^q})$; $\quad \log a\sqrt{a\sqrt{a}}$.

82. $\log \dfrac{a^2\sqrt[3]{a^2\sqrt{b^3}}}{a\sqrt[6]{b}}$; $\quad \log\sqrt{\dfrac{3a^2\sqrt{b}}{2b^2\sqrt{a}}}$; $\quad \log \dfrac{ab^2(a^3b)^3\sqrt[5]{a^3\sqrt{b}}}{(a+b)(a^2-b^2)}$; $\quad \log\sqrt[3]{\dfrac{(3a-b)^5\cdot\sqrt[4]{b^2+c}}{3a^2b\sqrt{c}}}$.

83. $\log x^{\log x}$; $\quad \log\sqrt[3]{5ab\sqrt{a+b}}$; $\quad \log\sqrt[5]{\dfrac{b^3c}{\sqrt{b^2-c^2}}}$; , $\quad \log\sqrt[4]{a\sqrt[3]{b\sqrt{c}}}$.

84. $\log\sqrt[4]{\dfrac{3}{7}}$; $\quad \log\sqrt[5]{\dfrac{11}{20}}$; $\quad \log\sqrt[3]{\left(\dfrac{2}{11}\right)^5}$; $\quad \log(\log 10^{mn})$; $\quad \log(\log\sqrt[m]{10^n})$.

85. $\log\sqrt{\dfrac{a\sqrt{a\sqrt{ab}}}{b\sqrt{a\sqrt{ab}}}}$; $\quad \log\sqrt[n]{a\sqrt[n]{b\sqrt[n]{c}}}$; $\quad \log\left(\dfrac{\sqrt[4]{b^3\sqrt{b\sqrt{b}}}}{\sqrt{b\sqrt[3]{b\sqrt{b}}}}\right)^5$.

Ridurre le somme in un unico logaritmo, qualunque sia la base:

86. $\log xy + \log\left(\dfrac{x}{y}-\dfrac{y}{x}\right)$; $\quad 3\log x + \dfrac{1}{2}\log y$; $\quad 4\log x - 3\log y$; $\quad \log(a^3-b^3)-\log(a-b)$.

87. $\dfrac{1}{2}\log 2(a-1) - \log 4 - \log(a+1)$; $\quad \dfrac{1}{2}\log(7a-2) - \dfrac{1}{2}\log(6a+1)$.

88. $2\log(5-a)+\log(35+3a)$; $\quad \log a+\dfrac{1}{x}\log a-\dfrac{1}{y}(\log a+\log b)$.

89. $\dfrac{2}{3}\log a-\left(\dfrac{3}{2}\log b+\dfrac{3}{4}\log c\right)$; $\quad \dfrac{1}{5}(2\log a+3\log b)-\dfrac{1}{2}[\log(a+b)+\log(a-b)]$.

90. $\dfrac{1}{3}\left[\log a+\dfrac{1}{3}\log(a-b)-2\left(\log b+\dfrac{1}{3}\log 3\right)\right]$; $\quad \log(a+b)+2\log a-\dfrac{1}{2}[\log(a-b)+3\log b]$.

91. $2\log 3-\dfrac{1}{3}\left(2\log 5+\dfrac{1}{2}\log 7\right)$; $\quad 2\log(x-y)-\dfrac{1}{2}\log(x+y)-\dfrac{1}{2}\log(x^2-xy+y^2)$.

Sapendo che(1) $\log a=1$, $\log b=\dfrac{3}{2}$, $\log c=-2$, *calcolare*:

92. $\log\dfrac{ab^3}{c^2}$; $\quad \log\left(\dfrac{a^3}{b}\right)^3$; $\quad \log\sqrt{abc}$; $\quad \log\sqrt[3]{\dfrac{ab}{c^2}}$; $\quad \log a\sqrt{a\sqrt{a}}$.

93. $\log\dfrac{a^3\sqrt{a^3b}\,\sqrt[3]{ab^2}}{\sqrt[3]{ab^4}}$; $\quad \log\dfrac{a^2b^3\sqrt{c^3}}{a\sqrt{b}\,\sqrt[3]{c^4}}$; $\quad \log\sqrt[4]{a\sqrt[3]{b\sqrt{c}}}$; $\quad \log(\sqrt[m]{a^n}\,\sqrt[p]{b^q})$; $\quad \log a^{\log b^2}$.

94. *Determinare* $\log x$ *nei seguenti casi*:

a) $x=\dfrac{a^2\sqrt{\operatorname{sen}\alpha}}{b^3\sqrt[4]{c}}$; *b)* $x=m^{-2}n^3$; *c)* $x=3a^{-3}\sqrt{\cos 2\alpha}$;

d) $x=\dfrac{1}{3}a^{-\frac{1}{2}}\sqrt[3]{4\operatorname{tg}^2\alpha}$; *e)* $x=\left(\dfrac{m}{n}\right)^{\frac{m}{n}}$; *f)* $x=\dfrac{a^{-2}\sqrt[3]{4\operatorname{sen}^2\alpha}}{5b^3}$;

g) $x=\sqrt{\dfrac{a}{\sqrt[3]{ab}}\sqrt{\dfrac{a}{b}}}$; *h)* $x=\sqrt{\dfrac{5a^2}{\sqrt[5]{\left(\dfrac{a}{b^2}\right)^2}}}$; *i)* $x=2^{\sqrt[3]{4}}$;

l) $x=\log(a^2)$; *m)* $x=(\sqrt{2})^{\sqrt{2}}$; *n)* $x=\log(\sqrt{2}^{\sqrt{2}})$;

o) $x=\log\left(\sqrt[3]{3}^{\sqrt[3]{3}}\right)$; *p)* $x=\log(10^{ab})$; *q)* $x=\dfrac{10\log a}{\log a^3}$;

r) $x=\log\sqrt{\log 5}$.

[... *b)* $3\log n-2\log m$; *c)* $\log 3+\dfrac{1}{2}\log\cos 2\alpha-3\log a$; *d)* $\dfrac{1}{3}(\log 4+2\log\operatorname{tg}\alpha)-\dfrac{1}{2}\log a-\log 3$; *e)* $\dfrac{m}{n}(\log m-\log n)$ *f)* $\dfrac{1}{3}(\log 4+2\log\operatorname{sen}\alpha)-(\log 5-3\log b-2\log a)$; *g)* $\dfrac{1}{12}(7\log a-5\log b)$; *h)* $\dfrac{1}{15}(5\log 5+8\log a+4\log b)$; ...; *l)* $\log 2+\log(\log a)$; *m)* ...; *n)* $\log(\log 2)-\dfrac{1}{2}\log 2$; *o)* $\log(\log 3)-\dfrac{2}{3}\log 3$; *p)* $\log a+\log b+\log(\log 10)$; *q)* $\log 10-\log 3$; *r)* $\log[\log(\log 5)-\log 2]$]

(1) In questo e nel successivo gruppo di esercizi, i logaritmi si intendono calcolati rispetto a una stessa base α, che viene sottointesa; così per es., $\log a$ sta per $\log_\alpha a$.

Completare le seguenti eguaglianze:

95. $4\log x - 3\log y = \log\ldots$; $\log(a^3 - b^3) - \log(a - b) = \log\ldots$; $\frac{1}{2}\log 2(a-1) - \log 4 - \log(a+1) = \log\ldots$.

96. $\log a + \frac{1}{x}\log a - \frac{1}{y}(\log a + \log b) = \log\ldots$; $\log(a+b) + 2\log a - \frac{1}{2}[\log(a-b) + 3\log b]$.

Applicando i teoremi inversi sui logaritmi, ricavare il valore della x dalle seguenti eguaglianze:

97. $\log x = \log a + \log b$; $\log x = \log a - \log b$. $\left[x = ab;\ x = \frac{a}{b}\right]$

98. $\log x = \frac{1}{2}\log a + \frac{1}{3}\log b$; $\log x = \frac{1}{2}\log a - \frac{1}{3}\log b$. $\left[x = \sqrt{a}\cdot\sqrt[3]{b};\ x = \frac{\sqrt{a}}{\sqrt[3]{b}}\right]$

99. $\log x = 2\log a + \frac{1}{2}\log b - \frac{2}{3}\log c$. $\left[x = \frac{a^2\sqrt{b}}{\sqrt[3]{c^2}}\right]$

100. $\log_p x = 1 - \frac{1}{2}\log_p a + \frac{2}{3}\log_p b$. $\left[x = \frac{p\sqrt[3]{b^2}}{\sqrt{a}}\right]$

101. $\log_p x = \frac{1}{2} + \frac{2}{3}\log_p a - \frac{1}{2}\log_p b$. $\left[x = \frac{\sqrt{p}\cdot\sqrt[3]{a^2}}{\sqrt{b}}\right]$

102. $\log x = 2\log a + \frac{1}{2}(\log b + \log c) - 5\log c$. $\left[x = \frac{a^2\sqrt{bc}}{c^5}\right]$

103. $\log x = \log a + \frac{1}{2}\left\{\log a + \frac{1}{4}\left[\log a + \frac{1}{3}\left(\log a + \frac{1}{2}\log a\right)\right]\right\}$. $\left[x = a\sqrt{a\sqrt[4]{a\sqrt[3]{a\sqrt{a}}}}\right]$

104. $\log x = \frac{1}{3}\log(a+b) - 2\log(a^2+b^2) + \frac{1}{2}\log a + \frac{1}{3}\log b$. $\left[x = \frac{\sqrt{a}\cdot\sqrt[3]{b(a+b)}}{(a^2+b^2)^2}\right]$

105. $\log_{10}x = 2 - \log_{10}b + \frac{1}{2}(2 - \log_{10}b) - \frac{1}{4}\left[2 - \log_{10}b - \frac{1}{2}(\log_{10}b - 1)\right]$. $\left[x = \frac{100}{b}\sqrt[8]{\frac{1000}{b}}\right]$

106. $\log_3 x = \log_3(a-b) - \frac{1}{2}\left[\log_3(a+b) - \frac{2}{3} + \frac{1}{2}\log_3(a-b) + 1\right]$. $\left[x = \frac{\sqrt[4]{(a-b)^3}}{\sqrt[6]{3}\cdot\sqrt{a+b}}\right]$

D) Funzioni logaritmiche

Determinare l'insieme di esistenza delle seguenti funzioni logaritmiche(1):

107. $f(x) = 1 - \log x$. $[x > 0]$

108. $f(x) = \log(3x-1) + 2\log(x+1)$. [Deve essere $3x - 1 > 0$ e $x + 1 > 0$. Si trova: $x > \frac{1}{3}$]

109. $f(x) = \log_x 2$. $[x > 0$, con $x \neq 1]$ **110.** $f(x) = \log\log(x+2)$. $[x > -1]$

(1) Per risolvere buona parte dei seguenti esercizi occorre la conoscenza delle disequazioni.