1. $\log(x-9) + \log x = \log 10.$ [10, la soluzione -1 non è accettabile]

2. $\log(2x+1) - \log 2 = \log(3-x).$ $\left[\frac{5}{4}\right]$

3. $\log 3 + \log x = \log 12;$ $\quad \log x + \frac{1}{2}\log 16 = \log(x+4).$ $\left[4; \frac{4}{3}\right]$

4. $\log(7-x) - \log(2x^2 - 11x) = -\log x.$ [6]

5. $2\log x = 3\log 4.$ [Si ha $\log x^2 = \log 4^3$, da cui $x^2 = 4^3$, $x = 8$]

6. $\log(x-16) = \log 105 - \log x;$ $\quad \log(1 - \sqrt{x+2}) = \frac{1}{2}\log(3x+7).$ [21; -2]

7. $\log x - \log(x-1) = \log 7 - \log 6;$ $\quad \log x + \log 4 - \log(x-1) = \log 5.$ [7; 5]

8. $\log(5-x) + \log 3 = \log 2 + \log(x-2);$ $\quad \log x - 2\log(x-1) = \log 4 - \log 9.$ $\left[\frac{19}{5}; 4\right]$

9. $\log(x-2) + \log(2x-3) = 2\log x;$ $\quad \frac{1}{3}\log x = 2\log a.$ $\left[6; x^{\frac{1}{3}} = a^2\right]$

10. $2\log(x-1) = 1 - \log 5.$ $\left[\text{Si ha } 1 = \log 10\text{, e quindi } (x-1)^2 = \frac{10}{5}\right]$

11. $\log x + \log(x+1) = 2\log(1-x);$ $\quad \log(x+5) - \log(x-5) = 2.$ $\left[x(x+1) = (1-x)^2; \frac{x+5}{x-5} = 100\right]$

12. $\log 2 + \log x = 2\log(4x-15);$ $\quad \log(35 - x^3) = 3\log(5-x).$ $\left[\frac{9}{2}; 2 \text{ e } 3\right]$

13. $\log 2x - \log(2x-1) = 3\log 2 - \log 7.$ [4]

14. $\log(x-1) - \log(x-2) = \log(x-4) - \log(x-6).$ [*Impossibile*]

15. $\log 2 + 2\log(3x-2) - \log 13 = \log(6x-4).$ [5]

16. $\log x + \log(2x-1) - \log(2x+5) = \log 3.$ [5]

17. $\log(x^2 - 7x + 110) = 2;$ $\quad \log(x^2 + 3x + 36) = 1 + \log(x+3).$ [2, 5; 1, 6]

18. $\log(x^2+3) - 2\log x - \log 2 = \log 4 - \log(x^2 - 3).$ [3]

19. $\log\sqrt{x+1} + \log\sqrt{x-1} = 2 - \log 2.$ $\left[\text{Si ha: } \sqrt{x+1}\cdot\sqrt{x-1} = \frac{100}{2}\right]$

20. $2\log(x-7) - \log(x+1) = 1;$ $\quad x\log x - \log x = \log 2.$ $\left[\frac{(x-7)^2}{x+1} = 10; 2\right]$

21. $\log x + \log(2x) + \log(4x) = -3;$ $\quad \log 16x - \log 2x + \log 3x = \log 9 + \log 4 - \log 6.$ $\left[\frac{1}{20}; \frac{1}{4}\right]$

22. $\log(x-1)+\log(x-2)-\log(x-3)=0,77815125...$
$\left[\text{Si ha } 0,77815125... = \log 6, \text{ e quindi } \frac{(x-1)(x-2)}{x-3}=6\right]$

23. $\log\sqrt{4x^2+3x+4}-\log\sqrt{x^2-x+1}=\frac{1}{2}$. $\left[\frac{3}{2}; \frac{2}{3}\right]$

24. $\frac{1}{2}\log(x^2+x+4)-\frac{1}{2}\log(2x^2+x+4)=\log 4-\log 5$. $\left[3; -\frac{12}{7}\right]$

25. $\log x+\log x^2+\log x^3+\log x^4=20$. $[100]$

26. $2(\log x)^2-5\log x+3=0$; $\quad 3\log x+\frac{2}{\log x}=5$. $[10 \text{ e } \sqrt{1000}; 10 \text{ e } \sqrt[3]{100}]$

27. $5\log x(\log x-1)=2(\log x+6)$. $\left[\frac{1}{10}; 100\cdot\sqrt[5]{100}\right]$

Con l'uso delle tavole logaritmiche, o del calcolatore, risolvere le seguenti **equazioni esponenziali**:

28. $5^x=4$; $\quad 3^{2x}=2$; $\quad 4^{x+1}=5$; $\quad 3,2^x=\sqrt{\frac{3}{4}}$.

29. $5^{2x}\cdot 3^{3x+1}=2025$; $\quad 3^{2x}\cdot 5^{4x-1}=20$. $\left[1; \frac{1}{\log 75}\right]$

30. $3^{3x}\cdot 3^{1-2x}=40$; $\quad \frac{3^x\cdot 5^x}{5}=32\cdot 2^x$. $\left[\frac{\log 40-\log 3}{\log 3}; \frac{5\log 2+\log 5}{\log 3+\log 5-\log 2}\right]$

31. $3\cdot 3^x+\frac{3^x}{3}-2^{2x}=\frac{2^{2x}}{2}$; $\quad 10+5^x=5^{x+1}$. $\left[\frac{\log 9-\log 20}{\log 3-\log 4}; \frac{\log 2,5}{\log 5}\right]$

32. $6\cdot 3^x-15\cdot 5^x+9\cdot 3^x=6\cdot 5^x$; $\quad \frac{2}{3}\cdot 3^x+3^{x+1}-110=0$. $\left[\frac{\log 7-\log 5}{\log 3-\log 5}; \frac{\log 30}{\log 3}\right]$

33. $3^{x+1}\cdot 3^{2x+3}=20\cdot 3^x$; $\quad 25^x-5^{x+1}+4=0$. $\left[\frac{\log 20}{2\log 3}-2; 0 \text{ e } \frac{\log 4}{\log 5}\right]$

34. $2^x+2^{x+1}+2^{x+2}=3^x+3^{x+1}+3^{x+2}$. $\left[\frac{\log 13-\log 7}{\log 2-\log 3}\right]$

35. $9^x-5\cdot 4^{x+1}=2\cdot 2^{2x}-4\cdot 3^{2x}$. $\left[\frac{\log 22-\log 5}{\log 9-\log 4}\right]$

36. $3^{2x}-7^{x+1}=2\cdot 7^{x-1}-9^{x+1}$. $\left[\frac{\log 51-\log 7-1}{\log 9-\log 7}\right]$

37. a) $4+\frac{2}{3^x-1}=\frac{5}{3^{x-1}}$. $[1; 0,2031]$ $\quad$ b) $\left(\frac{3}{5}\right)^{x-1}+\left(\frac{3}{5}\right)^{1-x}=2$. $[1]$

c) $\left(\frac{2}{3}\right)^x+9\cdot\left(\frac{3}{2}\right)^x=10$. $[0; -5,419]$

Risolvere le seguenti **equazioni esponenziali**:

38. $3^{|x^2-3x+2|}=9^{x+1}$; $\quad 3^{4\sqrt{x}}-4\cdot 3^{2\sqrt{x}}+3=0$. $\left[0 \text{ e } 5; 0 \text{ e } \frac{1}{4}\right]$

39. $25^{\sqrt{x-2}}-5\cdot 5^{\sqrt{x-2}}-500=0$; $\quad 4^{x-\sqrt{x^2-5}}-12\cdot 2^{x-1-\sqrt{x^2-5}}+8=0$. $\left[6; \frac{9}{4}, 3\right]$

49. $x^{\log x} - 4x^{-\log x} = 3.$

[Si ricava $x^{2\log x} - 4 = 3x^{\log x}$, da cui, posto $x^{\log x} = y$, si ricava $y^2 - 3y - 4 = 0$, da cui $y_1 = 4$, $y_2 = -1$, e quindi $x^{\log x} = 4$, ossia $(\log x)^2 = \log 4$, $\log x = \pm\sqrt{\log 4}$; da cui: $x = 10^{\pm\sqrt{\log 4}}$]

50. $3x^{\log x} + 100x^{-\log x} = 40; \qquad 2^{\log x} = 8; \qquad 5^{\log 2x} = 625.$ $\left[10, \frac{1}{10}, 10^{\pm\sqrt{1-\log 3}}; 10^3; 5 \cdot 10^3\right]$

51. $x^{3(4-3\log x)} = 1000; \qquad 5x^{\log x} + 2x^{-\log x} - 7 = 0.$ $[10 \text{ e } \sqrt[3]{10}; 1]$

52. $x^x = x^2; \qquad x^{2x} = x.$ $\left[2 \text{ e } 1; \frac{1}{2} \text{ e } 1\right]$

53. $\log_3 x - \log_9 x = 4; \qquad \log_4 x + \log_8 x = 3; \qquad \log_2 x - \log_8 x = 2.$ $\left[3^8; 2^{\frac{18}{5}}; 8\right]$

54. $\log_2 x + 3\log_8 \sqrt{x} = \frac{3}{2}; \qquad 4\log_2 x - 3\log_8 x = 6; \qquad 3\log_9 x + \log_3 x = 10.$ $[2; 4; 81]$

55. $x^{\log\sqrt{x}} = \sqrt{10}; \qquad \log_{125} x + \log_{25} x + \log_5 x = 11.$ $\left[10 \text{ e } \frac{1}{10}; 5^6\right]$

56. $\log \frac{8 - 2^{2x}}{4} - \log(3 + 4^{-2x}) = \log \frac{16}{49}; \qquad x^{\log x} = 16(6x^{\log\sqrt{x}} + 25).$ $[1, ...; 10^2; 10^{-2}]$

57. $\log_3 \sqrt{x-1} - \log_3 \sqrt{x-8} = 2\log_9 4 - 1.$ $[17]$

58. $\log_3 \sqrt{2x+1} - \log_3 \sqrt{3x+4} = 1 - 2\log_4 4.$ $\left[-\frac{1}{3}\right]$

59. $2\log_9 2 + \log_3 \sqrt{x+1} - \log_3 \sqrt{2x-7} = 3\log_{27} 2.$ $[8]$

60. $\log_a(ax) \cdot (\log_x a + 1) = -\log_{a^2} a; \qquad \frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3.$ $\left[\frac{1}{a^2} \text{ e } \frac{1}{\sqrt{a}}; \frac{1}{8} \text{ e } 2\right]$

61. $\sqrt{\log_2 x^4} + 4\log_2 \sqrt{\frac{2}{x}} = 2; \qquad \log^2 x^3 - 20\log\sqrt{x} + 1 = 0.$ $[1 \text{ e } 2; 10 \text{ e } \sqrt[9]{10}]$

62. $\log_2 x - 8\log_{x^2} 2 = 3; \qquad \log_{\sqrt{x}} 2 + 4\log_4 x^2 + 9 = 0.$ $\left[16 \text{ e } \frac{1}{2}; \frac{1}{4} \text{ e } \frac{1}{\sqrt[4]{2}}\right]$

63. $\log_x 3 \cdot \log_{\frac{x}{3}} 3 + \log_{\frac{x}{81}} 3 = 0; \qquad \log_x(ax) \cdot \log_a x = 1 + \log_x \sqrt{a}.$ $\left[9 \text{ e } \frac{1}{9}; a^{-\frac{1}{\sqrt{2}}} \text{ e } a^{\frac{1}{\sqrt{2}}}\right]$

64. $\log_3 a - \log_x a = \log_{\frac{x}{3}} a; \qquad \log_2 \log_2 \log_2 x = 0.$ $\left[3^{\frac{3+\sqrt{5}}{2}} \text{ e } 3^{\frac{3-\sqrt{5}}{2}}; 4\right]$

65. $\log_{\sqrt{2}} \log_2 \log_4 (x - 15) = 0; \qquad \log(\log_2 \log_3 \sqrt{x} + 1) = 0.$ $[31; 9]$

66. $\log_x \log_3 \log_x 2x^2 = 0; \qquad \log[3 + 2\log(1 + x)] = 0.$ $\left[2; -\frac{9}{10}\right]$

67. $\log(2^x + x - 4) = x(1 - \log 5); \qquad \log(x^2 - x - 6) - x = \log(x + 2) - 4.$ $[4; 4, ...]$

68. $2\log^5 x = 5\log x - 3\log^3 x.$ $\left[\frac{1}{10}; 1; 10\right]$

69. $\log_2 x + \log_3 x = 1; \qquad \log_2 x + \log_x 2 = 2.$ $\left[... \simeq \frac{153}{100}; 2\right]$

70. $\log_9 x + \log_{x^2} 3 = 1; \qquad \log_5 x + \log_x 5 = \frac{5}{2}.$ $[3; 25; \sqrt{5}]$

71. $\log_a x \log_b x = \log_a b; \qquad \log_{a^2} x + \log_{x^2} a = 1.$ $\left[b, 1b; a\right]$

72. $\log_3(4^x + 15 \cdot 2^x + 27) - 2\log_3(4 \cdot 2^x - 3) = 0.$ $[\log_2 3]$