MILLIKAN’S OIL-DROP EXPERIMENT

The quantization of electric charge

Robert A. Millikan
(March 22, 1868 – December 19, 1953)

was an American experimental physicist

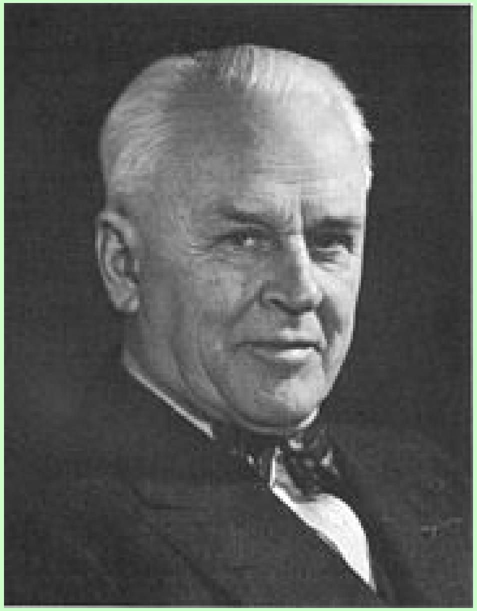
awarded the Nobel Prize for Physics

in 1923 for his measurement of

the elementary electronic charge

and for his work on the

photoelectric effect.



Starting in 1906, while a professor at the University of Chicago, Millikan worked on an oil-drop experiment, over the years.

He published his results in 1913.
Millikan and his then graduate student
Harvey Fletcher
performed the oil-drop experiment
to measure the charge of the electron,
and, as a result, the electron mass
and Avogadro’s number.

The brilliant idea

of using oil instead

of water seems to be

ascribed to Fletcher.

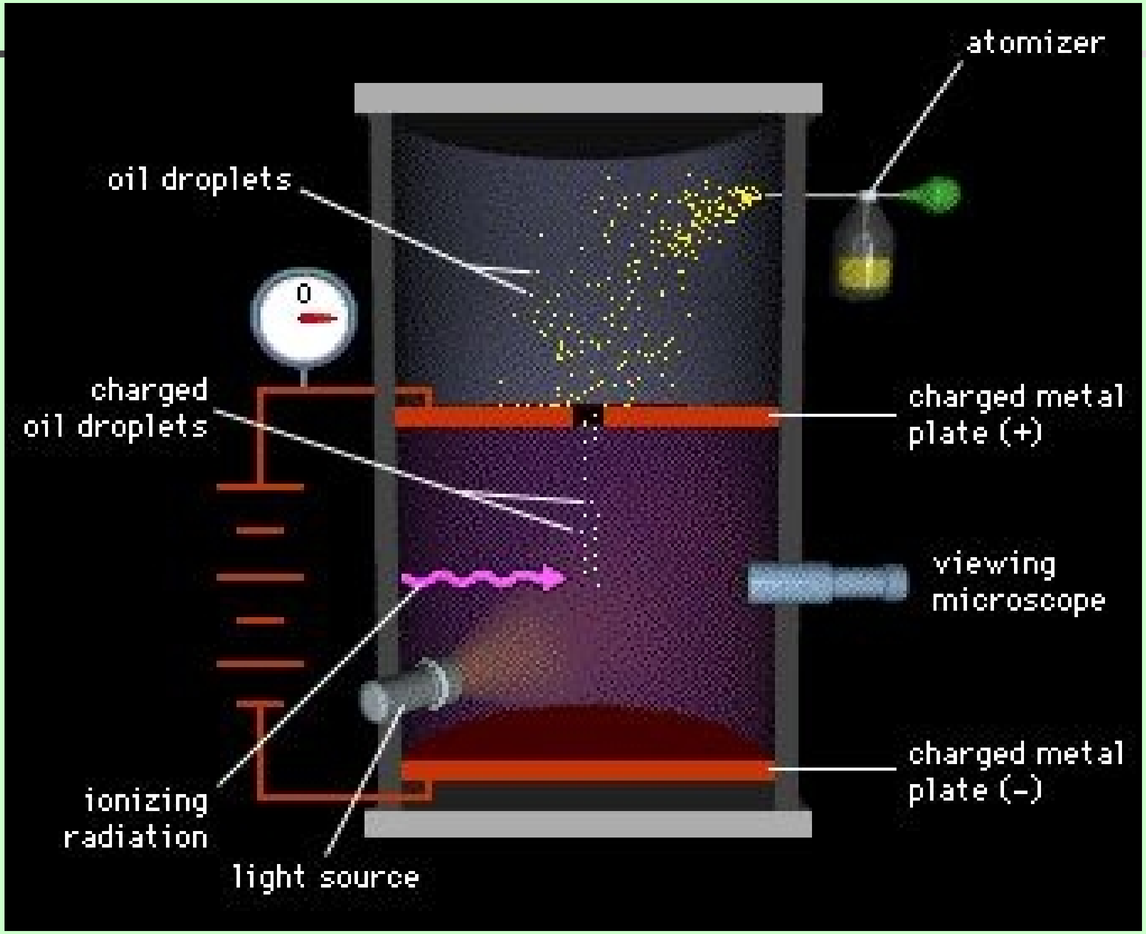
Oil drops evaporate

very slowly

and they can be observed

during their motion.

The oil-drop experiment was realized by using a capacitor with two parallel horizontal plates.



John Joseph Thomson had already discovered the charge-to-mass ratio (e/m) of the electron, in his famous experiment, performed in 1897.
He had proved that
the so called cathode rays,
observed in a vacuum tube,
were streams of particles
with a negative charge
which were named electrons.

However, the actual charge (e)
and mass (m)
values were unknown.
Therefore, if one
of these two values
were to be discovered,
the other could easily be calculated.

Oil drops are sprayed into the capacitor through a small perforation; most of them are charged by friction with the nozzle of the atomizer.

The motion of the drops can be observed through a microscope with a graduated scale that allows the experimenter to measure the velocity of the falling droplets.

https://www.youtube.com/watch?v=UFiPWv03f6g

We can divide the experiment into three phases.

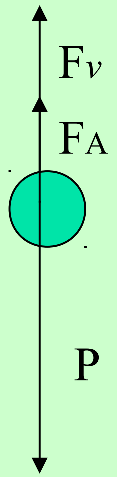
The first one enables us to calculate the radius of the drops and it is performed in the absence of an electric field.

Let’s analyze the experiment from a theoretical point of view.

The drops fall through the air between the plates of the capacitor.

A single drop is subjected to three forces:

- Gravity (P)
- Archimedes' force (F_A)
- Friction with air (F_v)



So the equation of motion is:

$$ma = P - F_A - F_v$$

The viscous drag force is described by Stokes's law:

$$F_v = 6\pi\eta rv$$

η is the coefficient of friction and 6πr is the geometric coefficient used for a spherical object:

$$ma = mg - V\rho_{air}g - 6\pi\eta rv$$

$$ma = V\rho_{oil}g - V\rho_{air}g - 6\pi\eta rv = Vg\left(\rho_{oil} - \rho_{air}\right) - 6\pi\eta rv$$

$$ma = V\rho g - 6\pi\eta rv$$

where V is the volume of the droplets.

The frictional force grows up with increasing velocity till it exactly balances the weight force, for a certain value of speed $v1$.

$$V\rho g = 6\pi\eta r v_1$$

So acceleration tends to zero, the drop falls with a constant velocity v_1, which can be measured. It is now possible to calculate the radius of the drop. η and ρ are known from previous experiments.

$$\boxed{v_1 = \frac{V\rho g}{6\pi\eta r} = \frac{4}{3}\pi r^3 \frac{\rho g}{6\pi\eta r} = \frac{2}{9}\frac{\rho g r^2}{\eta}}$$

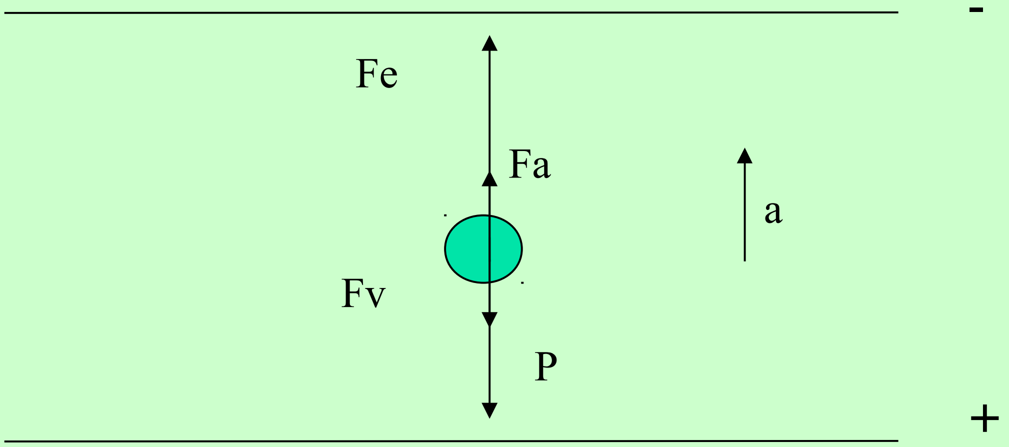
In the second phase, Millikan applied an **electric field** between the plates.

The droplets were randomly charged and they were pulled upwards or fell downwards depending on their charge and on the intensity and direction of the electric field.

Millikan observed a very high number of motions and obtained a lot of data.

We now consider
the case of a drop
with a positive charge
and
an electric potential
strong enough
to overcome
the other forces.

The drop is pulled upwards by the electric force.



The equation of motion is:

$$ma = F_e + F_A - F_v - P$$

For a given value of the velocity, v_2, the total force and the acceleration are equal to zero.

$$Fe + Fa - P = Fv$$

$$qE - \rho V g = 6\pi\eta r v_2$$

$$v_2 = \frac{qE - \rho V g}{6\pi\eta r} = \frac{qE}{6\pi\eta r} - v_1$$

It is possible to obtain the value of **q** from this equation.

But Millikan’s aim was to prove that electric charge is quantized, namely that there is an elementary charge and all charges are multiples of this value.

So he had to find the greatest common divisor for the many values of charges he was going to obtain.

The charge on a single electron is minimal and it would have been impossible to find an accurate value if the charges of oil-drops had contained too many electrons.

Cosmic rays from atmosphere or X-rays, purposely placed into the air in the capacitor, might produce an ionization of particles and slightly modify the charge of the oil-drops.

So we can perform a **third phase** of the experiment and measure the difference **Δq** that will amount to just a small number of electronic charges.

We conduct the same process as before and find a new terminal velocity.

$$v_3 = \frac{q'E - \rho V g}{6\pi\eta r} = \frac{q'E}{6\pi\eta r} - v_1$$

We calculate the difference v_3-v_2.

$$v_3 - v_2 = \frac{\Delta q E}{6\pi\eta r}$$

from this equation it is possible to obtain Δq, because all the other quantities are known or previously measured.

Millikan, working out thousands of measures over the years, could draw graphs that showed clearly how the charges were all multiples of a certain value.

Then he calculate the charge of an electron with a very high precision.

He obtained:

$e = 4.774(5) \times 10^{-10}$ statcoulombs,

that is

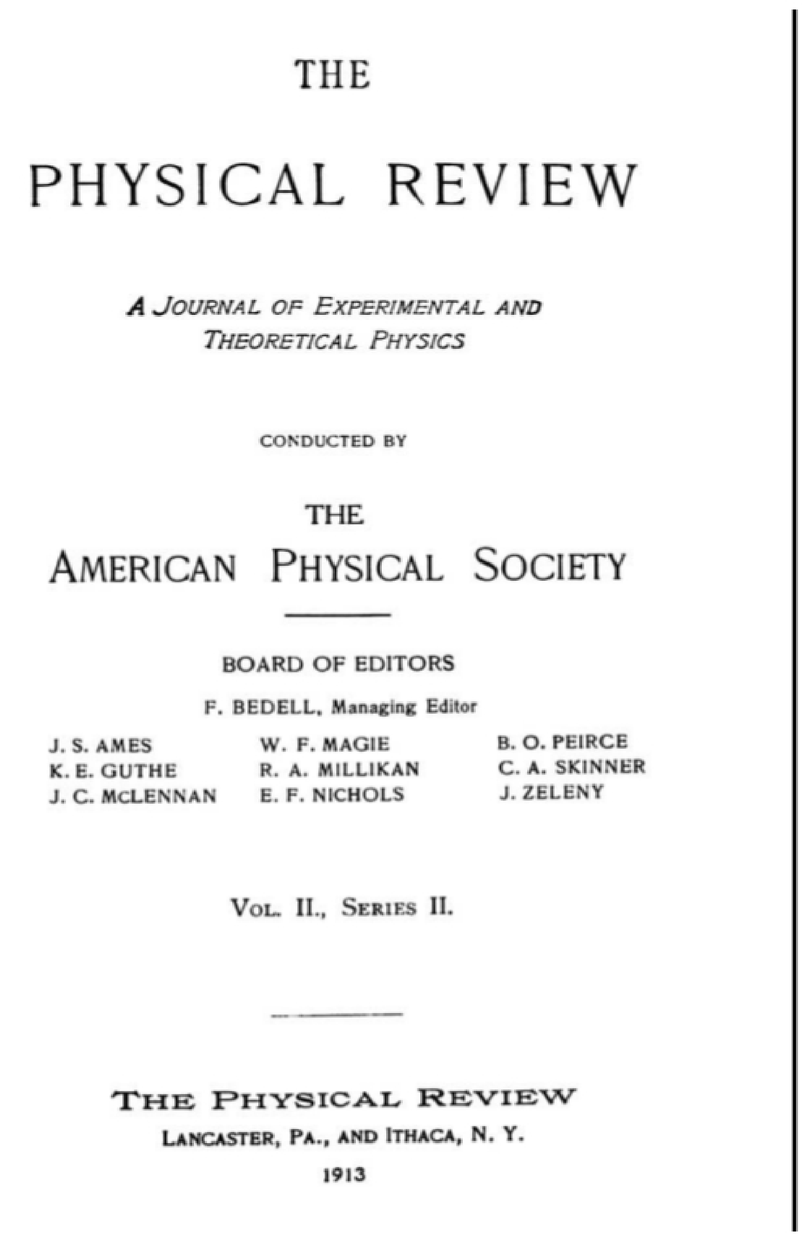
$e = 1.5924(17) \times 10^{-19}$ coulombs

with an error less than 1%.

Nowadays the accepted value for the electronic charge is:

e = 1.60217653(14) x 10^−19 coulomb.

Thereafter, it was also possible to calculate the mass of the electron, deriving it from Thomson’s result for e/m and to calculate a precise value of Avogadro’s number from Faraday’s constant.



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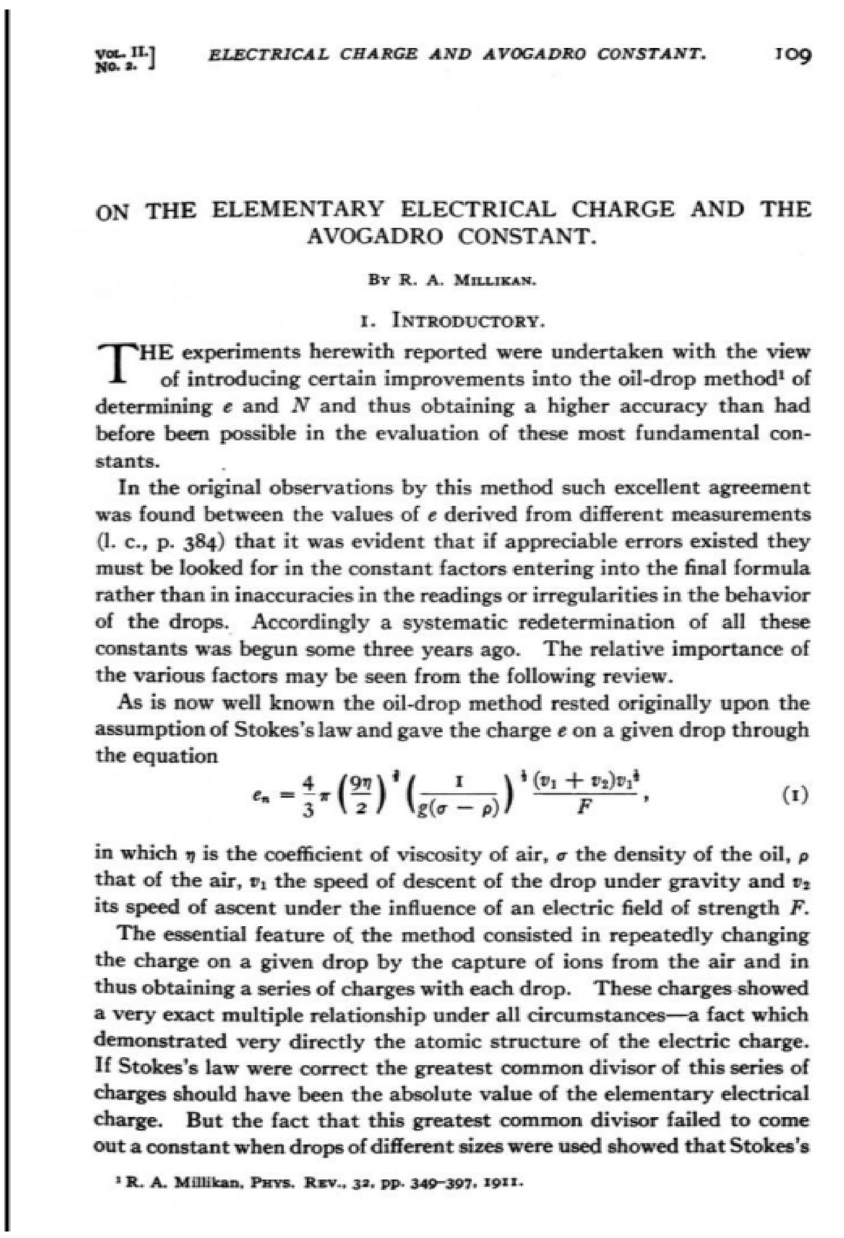


ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

BY R. A. MILLIKAN.

I. INTRODUCTORY.

THE experiments herewith reported were undertaken with the view of introducing certain improvements into the oil-drop method[1] of determining e and N and thus obtaining a higher accuracy than had before been possible in the evaluation of these most fundamental constants.

In the original observations by this method such excellent agreement was found between the values of e derived from different measurements (l. c., p. 384) that it was evident that if appreciable errors existed they must be looked for in the constant factors entering into the final formula rather than in inaccuracies in the readings or irregularities in the behavior of the drops. Accordingly a systematic redetermination of all these constants was begun some three years ago. The relative importance of the various factors may be seen from the following review.

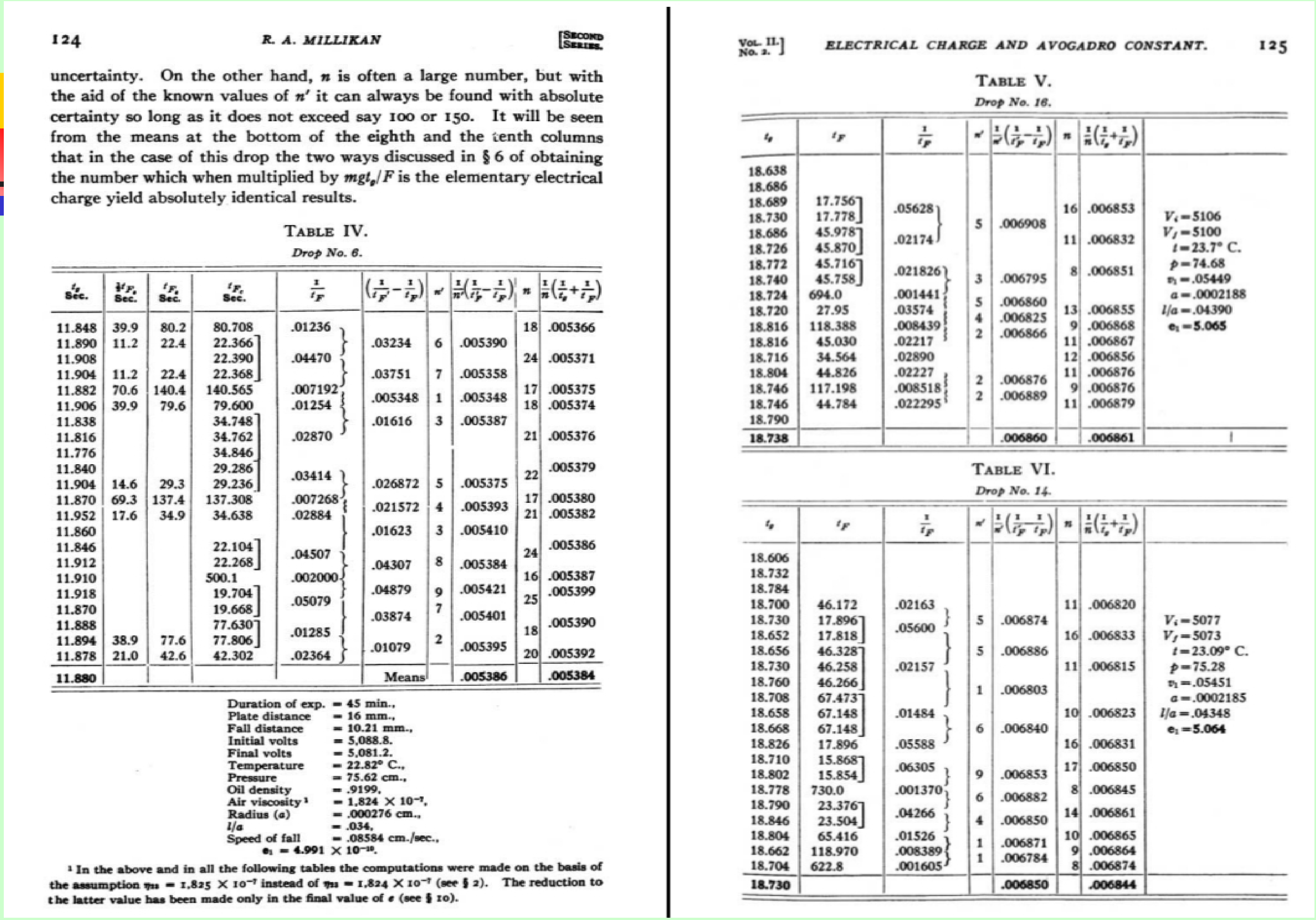
As is now well known the oil-drop method rested originally upon the assumption of Stokes's law and gave the charge e on a given drop through the equation

$$e_n = \frac{4}{3}\pi\left(\frac{9\eta}{2}\right)^{\frac{3}{2}}\left(\frac{1}{g(\sigma-\rho)}\right)^{\frac{1}{2}}\frac{(v_1+v_2)v_1^{\frac{1}{2}}}{F}, \qquad (1)$$

in which η is the coefficient of viscosity of air, σ the density of the oil, ρ that of the air, v_1 the speed of descent of the drop under gravity and v_2 its speed of ascent under the influence of an electric field of strength F.

The essential feature of the method consisted in repeatedly changing the charge on a given drop by the capture of ions from the air and in thus obtaining a series of charges with each drop. These charges showed a very exact multiple relationship under all circumstances—a fact which demonstrated very directly the atomic structure of the electric charge. If Stokes's law were correct the greatest common divisor of this series of charges should have been the absolute value of the elementary electrical charge. But the fact that this greatest common divisor failed to come out a constant when drops of different sizes were used showed that Stokes's

[1] R. A. Millikan, PHYS. REV., 32, pp. 349–397, 1911.



uncertainty. On the other hand, n is often a large number, but with the aid of the known values of n' it can always be found with absolute certainty so long as it does not exceed say 100 or 150. It will be seen from the means at the bottom of the eighth and the tenth columns that in the case of this drop the two ways discussed in § 6 of obtaining the number which when multiplied by mgt_g/F is the elementary electrical charge yield absolutely identical results.

TABLE IV.

Drop No. 6.

t_g Sec.	$\frac{1}{2}t_{F_s}$ Sec.	t_{F_s} Sec.	t_{F_c} Sec.	$\frac{1}{t_F}$	$\left(\frac{1}{t_{F'}}-\frac{1}{t_F}\right)$	n'	$\frac{1}{n'}\left(\frac{1}{t'_F}-\frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$
11.848	39.9	80.2	80.708	.01236				18	.005366
11.890	11.2	22.4	22.366		.03234	6	.005390		
11.908			22.390	.04470				24	.005371
11.904	11.2	22.4	22.368		.03751	7	.005358		
11.882	70.6	140.4	140.565	.007192	.005348	1	.005348	17	.005375
11.906	39.9	79.6	79.600	.01254				18	.005374
11.838			34.748		.01616	3	.005387		
11.816			34.762	.02870				21	.005376
11.776			34.846						
11.840			29.286	.03414				22	.005379
11.904	14.6	29.3	29.236		.026872	5	.005375		
11.870	69.3	137.4	137.308	.007268	.021572	4	.005393	17	.005380
11.952	17.6	34.9	34.638	.02884				21	.005382
11.860					.01623	3	.005410		
11.846			22.104	.04507				24	.005386
11.912			22.268		.04307	8	.005384		
11.910			500.1	.002000				16	.005387
11.918			19.704	.05079	.04879	9	.005421	25	.005399
11.870			19.668		.03874	7	.005401		
11.888			77.630	.01285				18	.005390
11.894	38.9	77.6	77.806		.01079	2	.005395		
11.878	21.0	42.6	42.302	.02364				20	.005392
11.880					Means		.005386		.005384

Duration of exp. = 45 min.,
Plate distance = 16 mm.,
Fall distance = 10.21 mm.,
Initial volts = 5,088.8.
Final volts = 5,081.2.
Temperature = 22.82° C.,
Pressure = 75.62 cm.,
Oil density = .9199.
Air viscosity[1] = $1{,}824 \times 10^{-7}$,
Radius (a) = .000276 cm.,
l/a = .034.
Speed of fall = .08584 cm./sec.,
$e_1 = 4.991 \times 10^{-10}$.

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[1] In the above and in all the following tables the computations were made on the basis of the assumption $\eta_{23} = 1{,}825 \times 10^{-7}$ instead of $\eta_{23} = 1{,}824 \times 10^{-7}$ (see § 2). The reduction to the latter value has been made only in the final value of e (see § 10).

TABLE V.

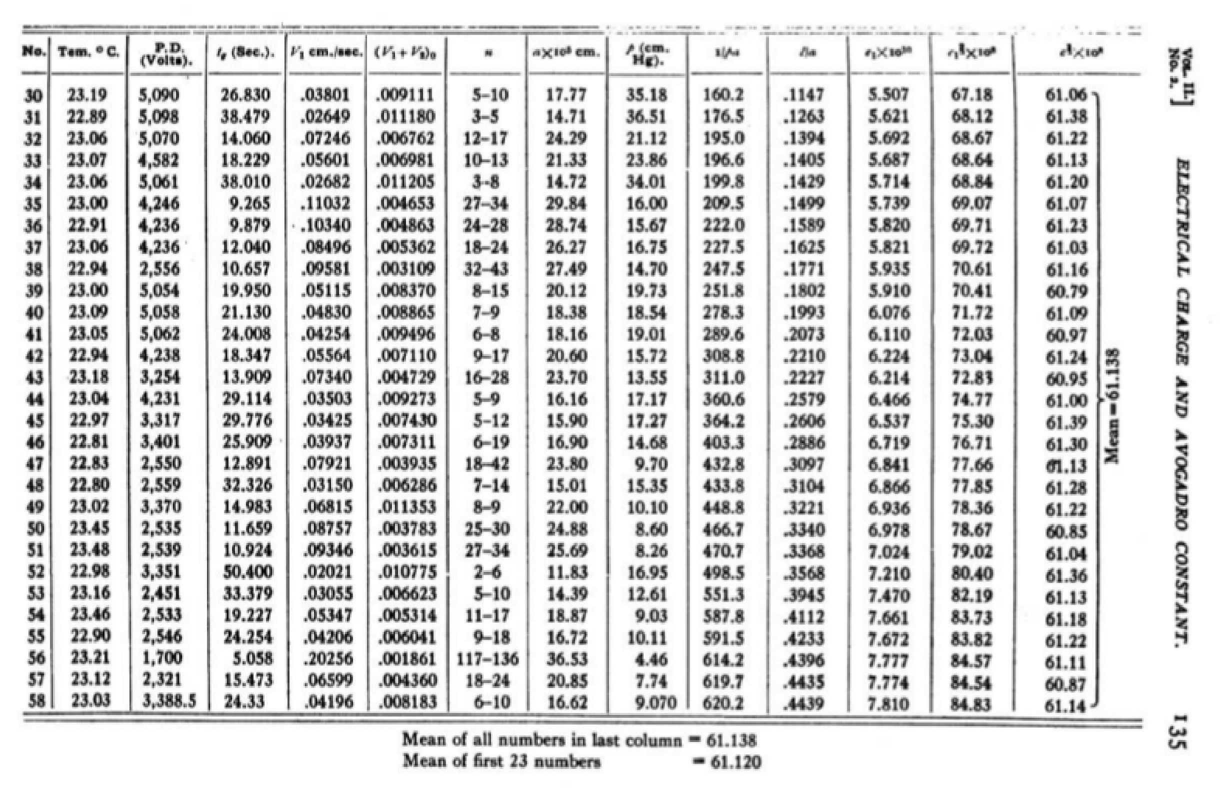
Drop No. 16.

t_g	t_F	$\frac{1}{t_F}$	n'	$\frac{1}{n'}\left(\frac{1}{t'_F}-\frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$	
18.638							
18.686							
18.689	17.756	.05628			16	.006853	
18.730	17.778		5	.006908			$V_i = 5106$
18.686	45.978	.02174			11	.006832	$V_f = 5100$
18.726	45.870						$t = 23.7°$ C.
18.772	45.716	.021826			8	.006851	$p = 74.68$
18.740	45.758		3	.006795			$v_1 = .05449$
18.724	694.0	.001441	5	.006860			$a = .0002188$
18.720	27.95	.03574	4	.006825	13	.006855	$l/a = .04390$
18.816	118.388	.008439	2	.006866	9	.006868	$e_1 = 5.065$
18.816	45.030	.02217			11	.006867	
18.716	34.564	.02890			12	.006856	
18.804	44.826	.02227	2	.006876	11	.006876	
18.746	117.198	.008518	2	.006889	9	.006876	
18.746	44.784	.022295			11	.006879	
18.790							
18.738				.006860		.006861	

TABLE VI.

Drop No. 14.

t_g	t_F	$\frac{1}{t_F}$	n'	$\frac{1}{n'}\left(\frac{1}{t'_F}-\frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$	
18.606							
18.732							
18.784							
18.700	46.172	.02163			11	.006820	
18.730	17.896	.05600	5	.006874			$V_i = 5077$
18.652	17.818				16	.006833	$V_f = 5073$
18.656	46.328		5	.006886			$t = 23.09°$ C.
18.730	46.258	.02157			11	.006815	$p = 75.28$
18.760	46.266						$v_1 = .05451$
18.708	67.473		1	.006803			$a = .0002185$
18.658	67.148	.01484			10	.006823	$l/a = .04348$
18.668	67.148		6	.006840			$e_1 = 5.064$
18.826	17.896	.05588			16	.006831	
18.710	15.868	.06305			17	.006850	
18.802	15.854		9	.006853			
18.778	730.0	.001370	6	.006882	8	.006845	
18.790	23.376	.04266			14	.006861	
18.846	23.504		4	.006850			
18.804	65.416	.01526	1	.006871	10	.006865	
18.662	118.970	.008389	1	.006784	9	.006864	
18.704	622.8	.001605			8	.006874	
18.730				.006850		.006844	



No.	Tem. °C.	P.D. (Volts).	t_g (Sec.).	V_1 cm./sec.	$(V_1+V_2)_0$	n	$a\times10^5$ cm.	p (cm. Hg).	$1/pa$	l/a	$e_1\times10^{10}$	$e_1^{2/3}\times10^8$	$e^{2/3}\times10^8$
30	23.19	5,090	26.830	.03801	.009111	5–10	17.77	35.18	160.2	.1147	5.507	67.18	61.06
31	22.89	5,098	38.479	.02649	.011180	3–5	14.71	36.51	176.5	.1263	5.621	68.12	61.38
32	23.06	5,070	14.060	.07246	.006762	12–17	24.29	21.12	195.0	.1394	5.692	68.67	61.22
33	23.07	4,582	18.229	.05601	.006981	10–13	21.33	23.86	196.6	.1405	5.687	68.64	61.13
34	23.06	5,061	38.010	.02682	.011205	3–8	14.72	34.01	199.8	.1429	5.714	68.84	61.20
35	23.00	4,246	9.265	.11032	.004653	27–34	29.84	16.00	209.5	.1499	5.739	69.07	61.07
36	22.91	4,236	9.879	.10340	.004863	24–28	28.74	15.67	222.0	.1589	5.820	69.71	61.23
37	23.06	4,236	12.040	.08496	.005362	18–24	26.27	16.75	227.5	.1625	5.821	69.72	61.03
38	22.94	2,556	10.657	.09581	.003109	32–43	27.49	14.70	247.5	.1771	5.935	70.61	61.16
39	23.00	5,054	19.950	.05115	.008370	8–15	20.12	19.73	251.8	.1802	5.910	70.41	60.79
40	23.09	5,058	21.130	.04830	.008865	7–9	18.38	18.54	278.3	.1993	6.076	71.72	61.09
41	23.05	5,062	24.008	.04254	.009496	6–8	18.16	19.01	289.6	.2073	6.110	72.03	60.97
42	22.94	4,238	18.347	.05564	.007110	9–17	20.60	15.72	308.8	.2210	6.224	73.04	61.24
43	23.18	3,254	13.909	.07340	.004729	16–28	23.70	13.55	311.0	.2227	6.214	72.83	60.95
44	23.04	4,231	29.114	.03503	.009273	5–9	16.16	17.17	360.6	.2579	6.466	74.77	61.00
45	22.97	3,317	29.776	.03425	.007430	5–12	15.90	17.27	364.2	.2606	6.537	75.30	61.39
46	22.81	3,401	25.909	.03937	.007311	6–19	16.90	14.68	403.3	.2886	6.719	76.71	61.30
47	22.83	2,550	12.891	.07921	.003935	18–42	23.80	9.70	432.8	.3097	6.841	77.66	61.13
48	22.80	2,559	32.326	.03150	.006286	7–14	15.01	15.35	433.8	.3104	6.866	77.85	61.28
49	23.02	3,370	14.983	.06815	.011353	8–9	22.00	10.10	448.8	.3221	6.936	78.36	61.22
50	23.45	2,535	11.659	.08757	.003783	25–30	24.88	8.60	466.7	.3340	6.978	78.67	60.85
51	23.48	2,539	10.924	.09346	.003615	27–34	25.69	8.26	470.7	.3368	7.024	79.02	61.04
52	22.98	3,351	50.400	.02021	.010775	2–6	11.83	16.95	498.5	.3568	7.210	80.40	61.36
53	23.16	2,451	33.379	.03055	.006623	5–10	14.39	12.61	551.3	.3945	7.470	82.19	61.13
54	23.46	2,533	19.227	.05347	.005314	11–17	18.87	9.03	587.8	.4112	7.661	83.73	61.18
55	22.90	2,546	24.254	.04206	.006041	9–18	16.72	10.11	591.5	.4233	7.672	83.82	61.22
56	23.21	1,700	5.058	.20256	.001861	117–136	36.53	4.46	614.2	.4396	7.777	84.57	61.11
57	23.12	2,321	15.473	.06599	.004360	18–24	20.85	7.74	619.7	.4435	7.774	84.54	60.87
58	23.03	3,388.5	24.33	.04196	.008183	6–10	16.62	9.070	620.2	.4439	7.810	84.83	61.14

Mean = 61.138

Mean of all numbers in last column = 61.138
Mean of first 23 numbers = 61.120

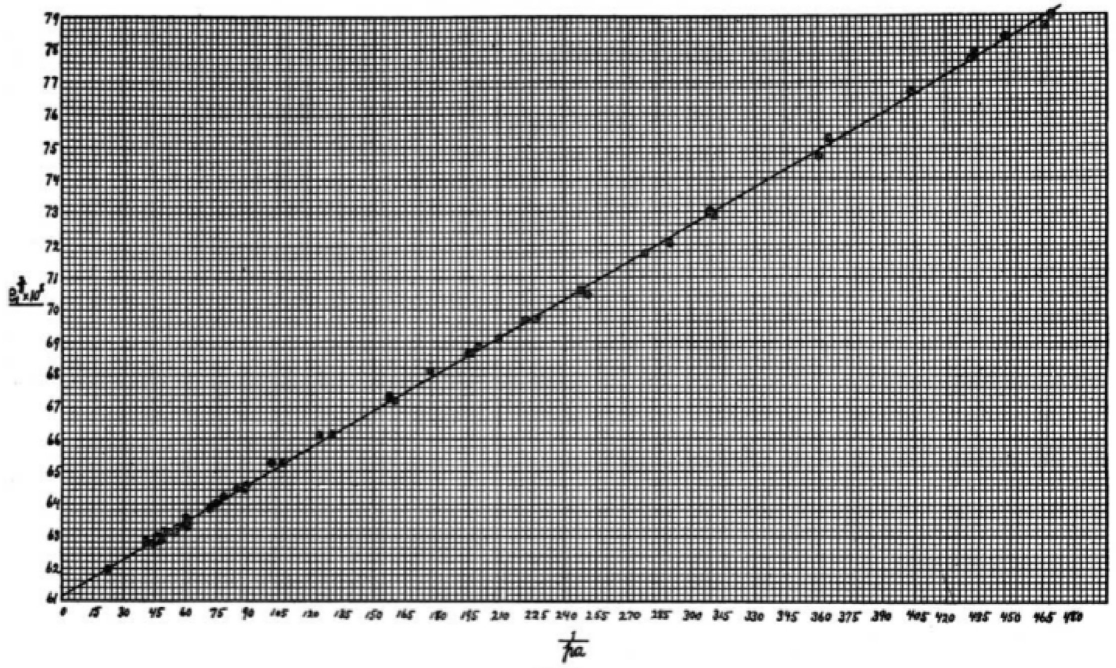
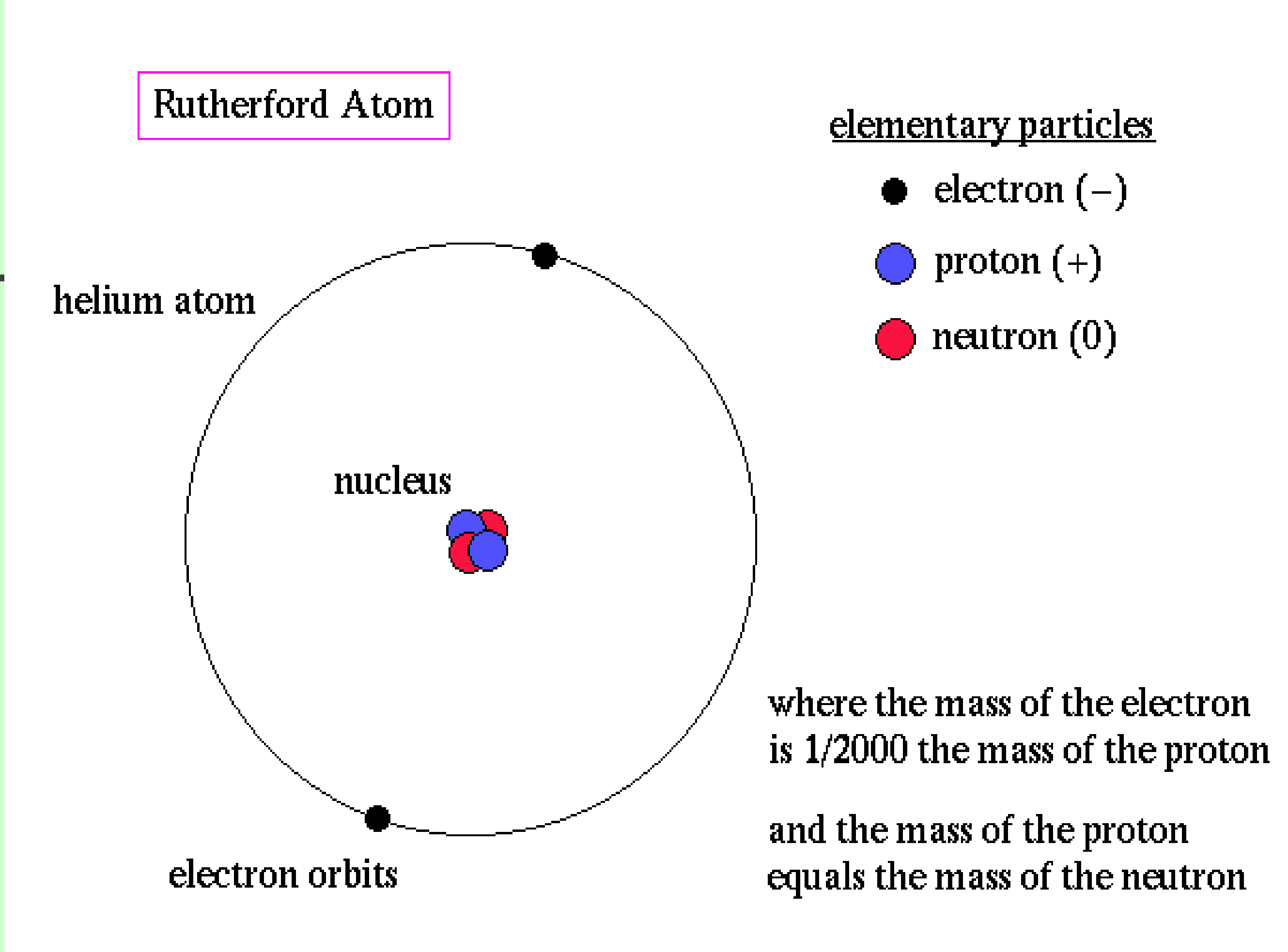


Fig. 2.

Thomson and Millikan verified that atoms contains charged particles called electrons and these electrons carry the “electricity quantum”. Rutherford had proved that electrons orbit around a small positive center and his model was the best description at that time of atomic structure but there were some unsolved problems...



where the mass of the electron is 1/2000 the mass of the proton

and the mass of the proton equals the mass of the neutron

Neutron will be discovered in 1932

The End