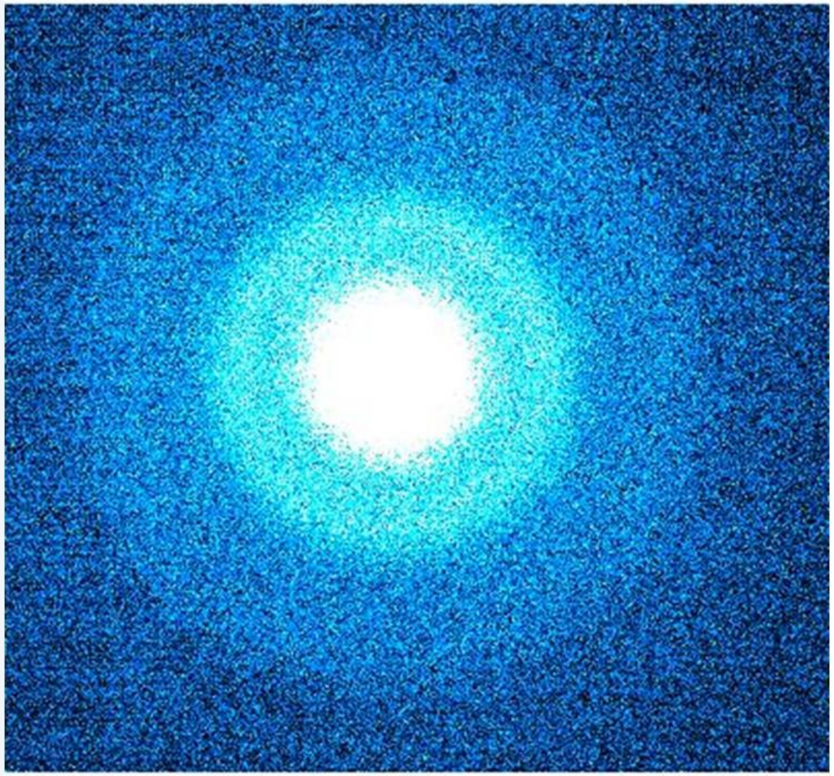
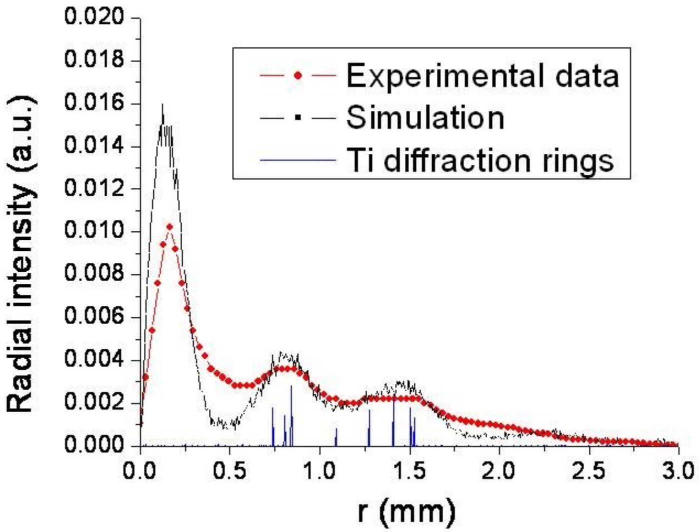
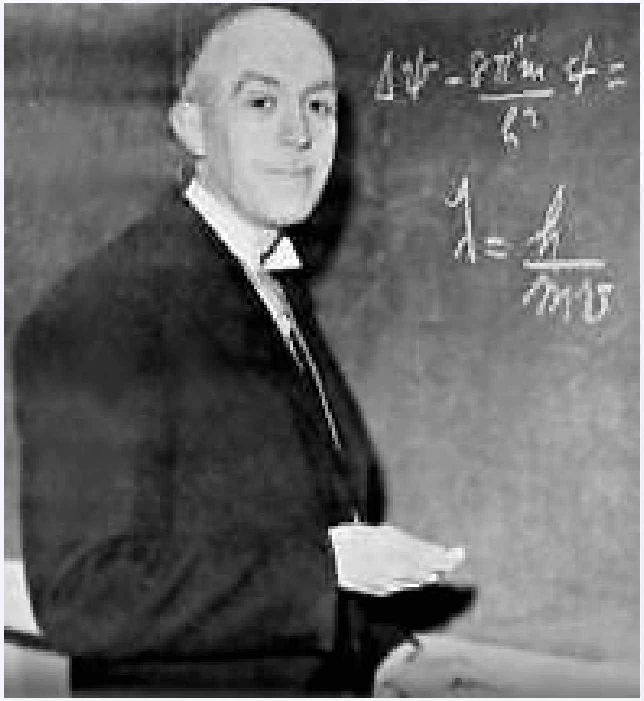
ELECTRON DIFFRACTION





In 1924 the French noble man Louis Victor prince De Broglie advanced a risky hypothesis. Matter could have an undulatory aspect just like electromagnetic waves have a corpuscular aspect.



He combined the relativistic expression for the photon momentum

$$E = cp$$

with the quantum of action

$$E = h\nu$$

This generalization
presupposed a complete symmetry
between radiation and matter.
It is easy to find an expression
for the wavelength
from the previous equations:

$$\lambda = \frac{c}{v} = \frac{h}{p}$$

If we apply this equation to a particle the expression becomes:

$$\lambda = \frac{h}{p} = \frac{h}{m_0 V} \cdot \sqrt{1 - \frac{V^2}{c^2}}$$

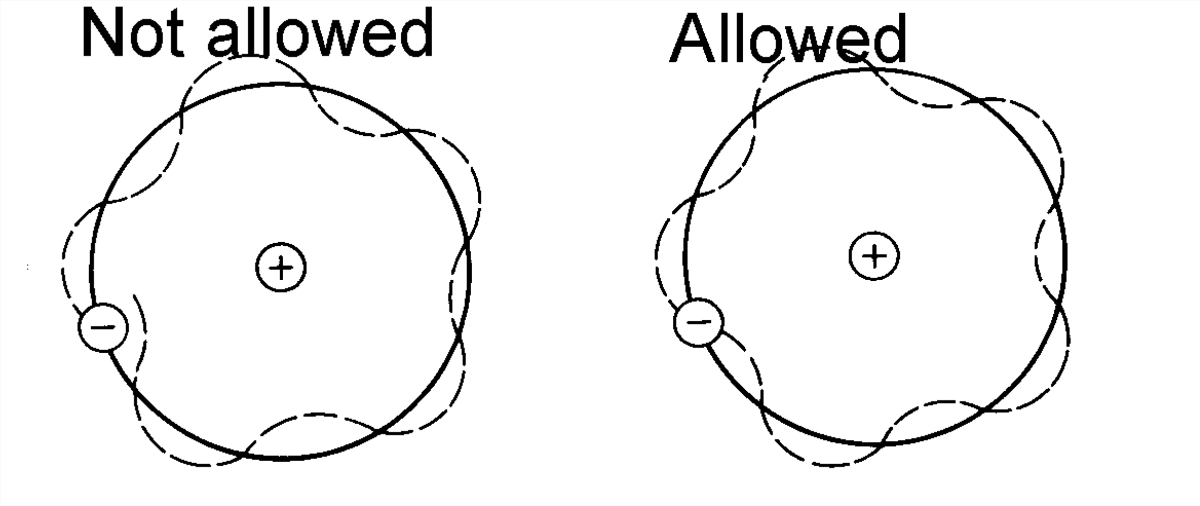
If the ratio V/c is small

the relation is simpler

$$\lambda = \frac{h}{p} = \frac{h}{m_0 V}$$

De Broglie's hypothesis
was not initially accepted.
Someone mocked it
and
called it
“La Comédie Française”

Actually De Broglie's wavelength
was in accordance with Bohr's atomic model.
If we hypothesize that
an electron has undulatory
behaviour in his orbital motion,
the associated wave must be a standing wave.
Otherwise there would be destructive interference.



The relation between the wavelength of an electron in a standing wave and the orbit radius is necessarily

$$2\pi r = n\lambda$$

But in Bohr's theory the angular momentum is quantized

$$mVr = n\frac{h}{2\pi}$$

If we compare the two equations
we instantly find:

$$\lambda = \frac{h}{mV}$$

that is the De Broglie's wavelength

If we try to calculate this wavelength
for a tennis ball,
with m = 50 g
and V = 40 m/s
we find

$$\lambda \sim 3{,}3 \cdot 10^{-34}\, m$$

that is certainly not observable!

But if we consider an electron

with a speed of about $10^7 \frac{m}{s}$

the wavelegth is

approximately 1 angstrom,

similar to X-rays wavelength.

So if we want to perform an experiment
to verify the undulatory nature of
electrons we can use
the same methods used for X-rays.
In particular, we can show
that electrons are subjected to a
typical wave phenomenon
such as DIFFRACTION.

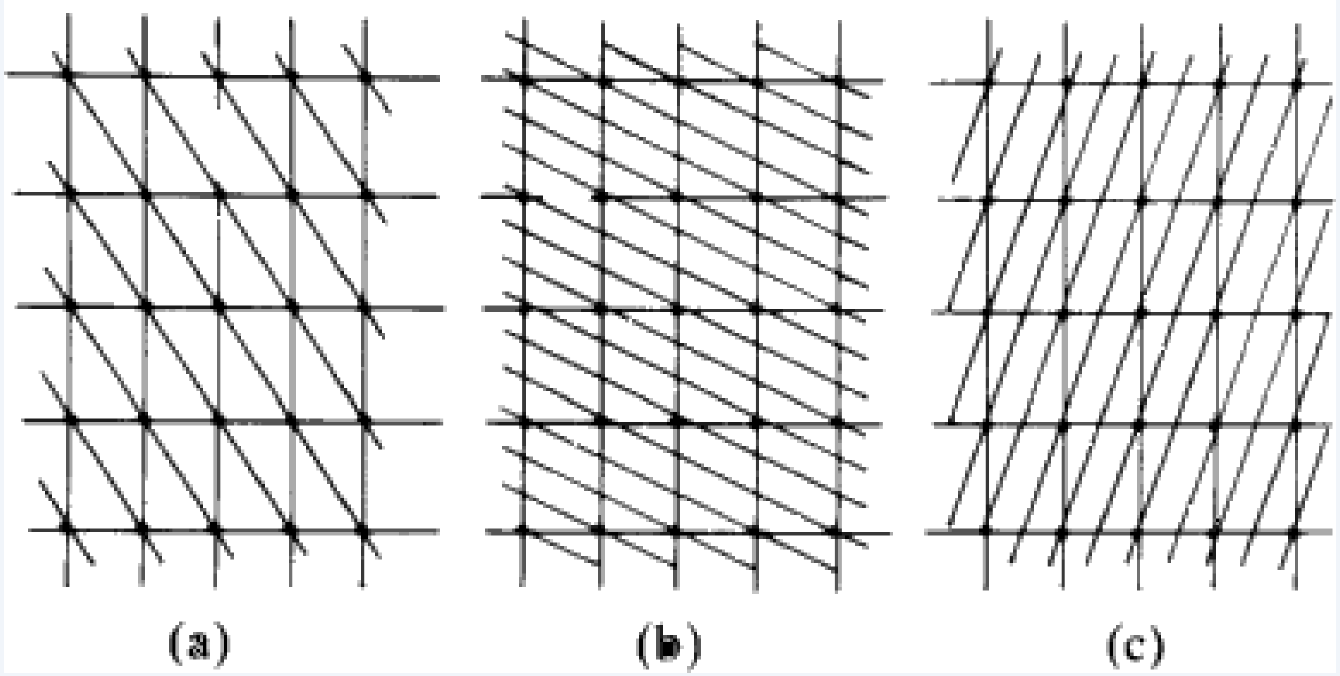
The brothers William Henry and William Lawrence Bragg discovered that crystalline solids produced surprising patterns of reflected X-rays.

They found out that these crystals, at certain specific wavelengths and incident angles, produced intense peaks of reflected radiation.

They founded the new science of X-ray crystallography, the analysis of crystal structure using X-ray diffraction.

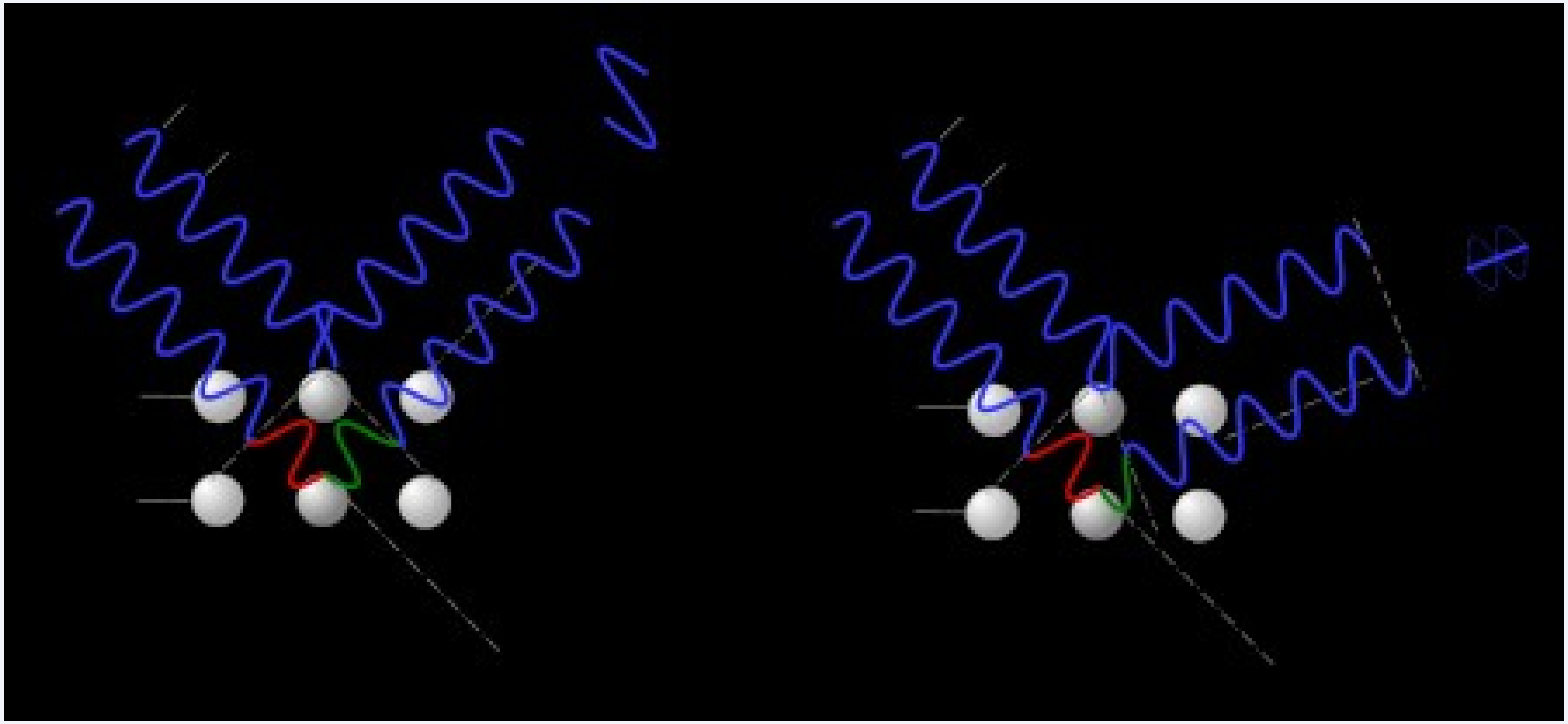
Crystals were an ideal diffraction
grating for X-rays,
because they have a regular
microscopic structure and
the distances among atoms are
comparable
to X-rays wavelength

The structure of a crystal lattice shows many parallel atomic planes



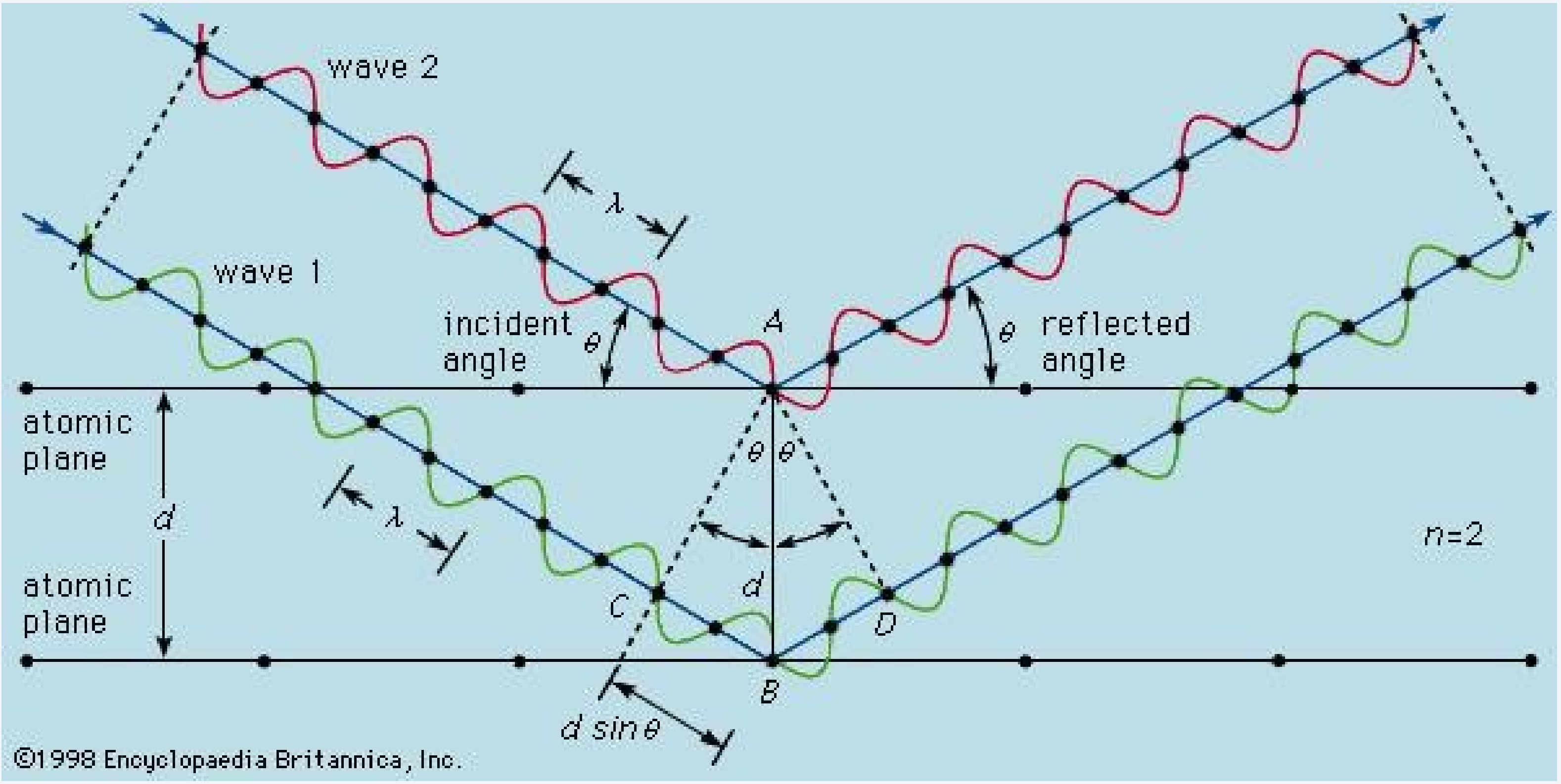
In this case we have three different possible distances between the planes.

X-rays interact with atoms in crystal



The phase shift causes constructive or destructive interference

The difference between the paths of the incident wave and of the scattered wave is 2d sin θ.



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The relation between the distance d of the atomic planes and the angle θ of incidence, in order to have constructive interference, is known as Bragg's law:

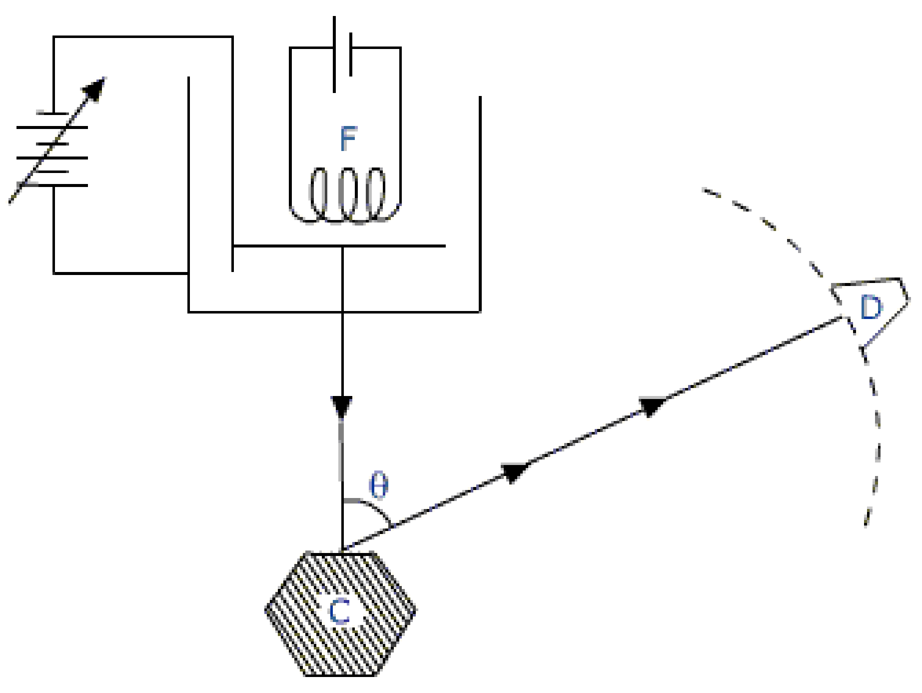
$$2d \sin\theta = n\lambda$$

Bragg's law can be used
in an experiment of diffraction
to measure the plane separation d,
if you know the wavelength,
or conversely
the wavelength if
you know d.

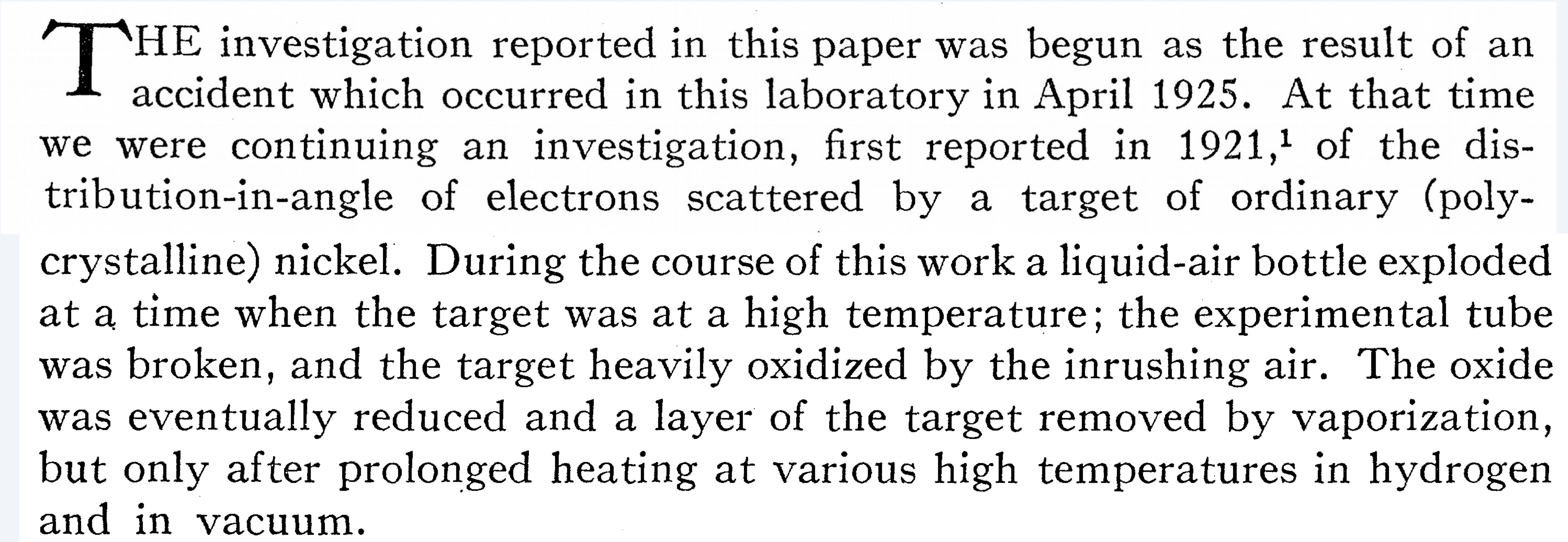
In 1927 there were two experimental
confirmations of
De Broglie's hypothesis.
The first one was due
to the American physicists
Clinton Davisson and Lester Germer,
the second one was obtained
by George Paget Thomson,
J.J. Thomson's son.

Davisson's and Germer's results
were accidentally found.
They were performing experiments
on electrons scattering
by a target of nichel.
The apparatus was similar to
the one used by Rutherford
in the gold foil experiment.
An accident occured.

The experimental arrangement to perform this kind of experiment is a crystal spectrometer

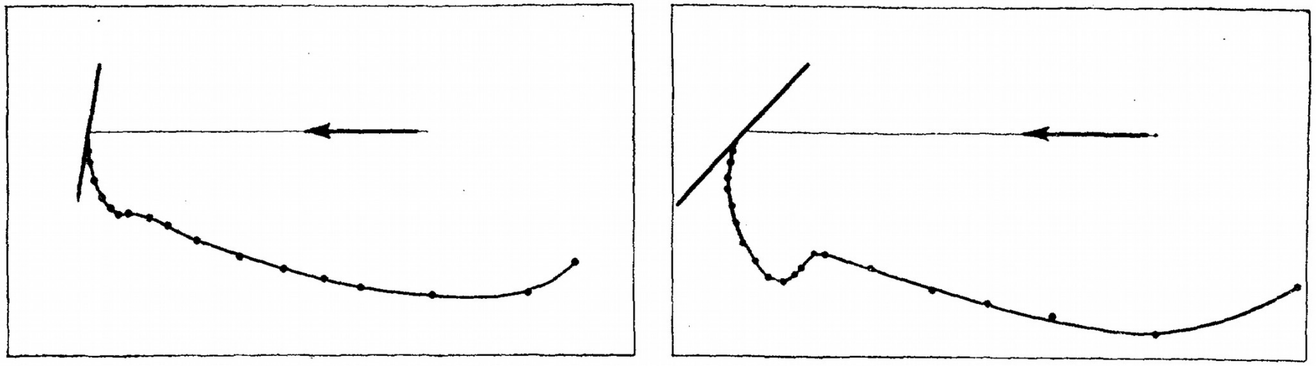


Let's read their own words.

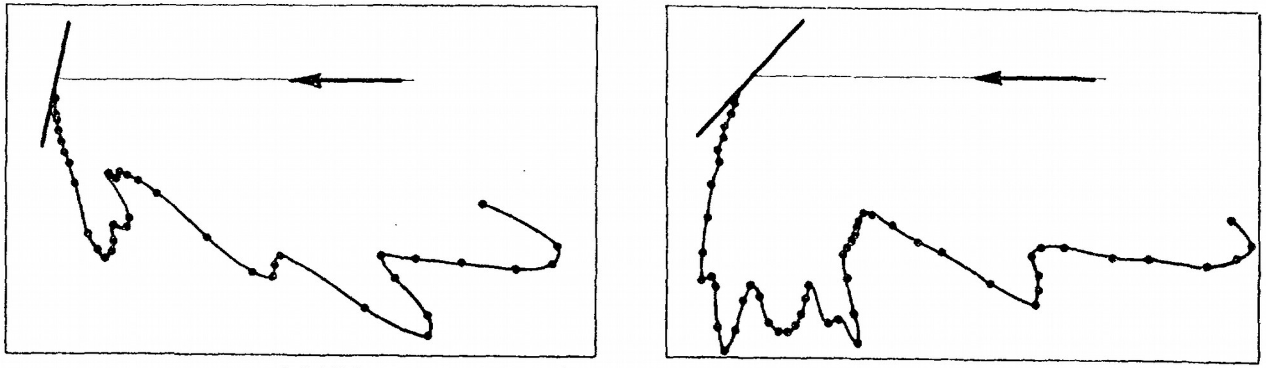


THE investigation reported in this paper was begun as the result of an accident which occurred in this laboratory in April 1925. At that time we were continuing an investigation, first reported in 1921,[1] of the distribution-in-angle of electrons scattered by a target of ordinary (polycrystalline) nickel. During the course of this work a liquid-air bottle exploded at a time when the target was at a high temperature; the experimental tube was broken, and the target heavily oxidized by the inrushing air. The oxide was eventually reduced and a layer of the target removed by vaporization, but only after prolonged heating at various high temperatures in hydrogen and in vacuum.

When the experiments were continued it was found that the distribution-in-angle of the scattered electrons had been completely changed. Specimen curves exhibiting this alteration are shown in Fig. 1. These curves are all for a bombarding potential of 75 volts. The electron beam is incident on the target from the right, and the intensities of scattering in different directions are proportional to the vectors from the point of bombardment to the curves. The upper curves (for different angles of incidence) are characteristic of the target prior to the accident.



SCATTERING OF 75 VOLT ELECTRONS FROM
A BLOCK OF NICKEL (MANY SMALL CRYSTALS)



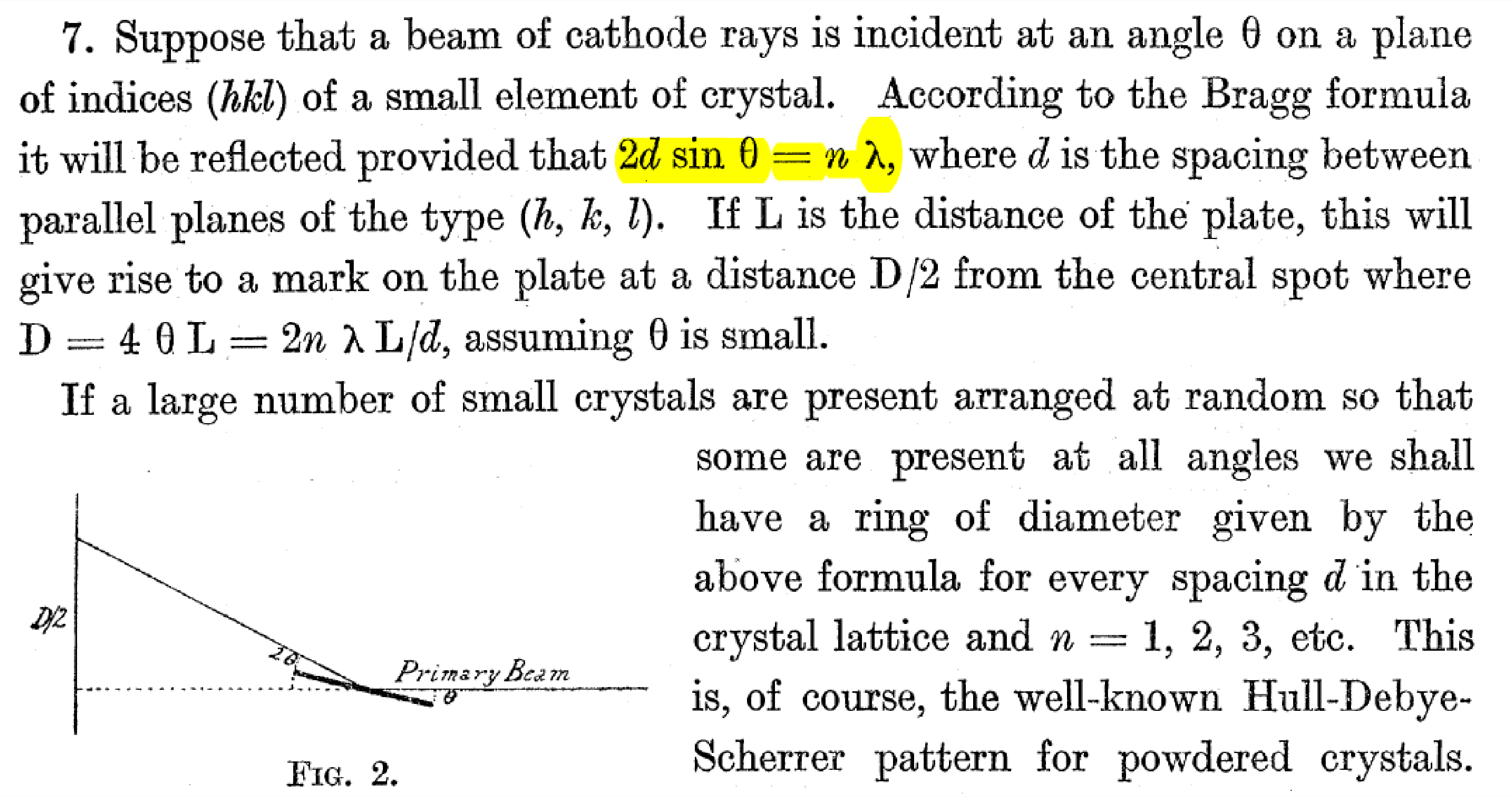
SCATTERING OF 75 VOLT ELECTRONS FROM
SEVERAL LARGE NICKEL CRYSTALS

Fig. 1. Scattering curves from nickel before and after crystal growth had occurred.

The most striking characteristic of these beams is a one to one correspondence, presently to be described, which the strongest of them bear to the Laue beams that would be found issuing from the same crystal if the incident beam were a beam of x-rays. Certain others appear to be analogues, not of Laue beams, but of optical diffraction beams from plane reflection gratings—the lines of these gratings being lines or rows of atoms in the surface of the crystal. Because of these similarities between the scattering of electrons by the crystal and the scattering of waves by three- and two-dimensional gratings a description of the occurrence and behavior of the electron diffraction beams in terms of the scattering of an equivalent wave radiation by the atoms of the crystal, and its subsequent interference, is not only possible, but most simple and natural. This involves the association of a wave-length with the incident electron beam, and this wave-length turns out to be in acceptable agreement with the value h/mv of the undulatory mechanics, Planck's action constant divided by the momentum of the electron.

In 1926-27 G.P. Thomson
performed
an experiment
following a method
similar to the one
you will use
in your experiment
in the Physics Lab

Let's read a part of his article



7. Suppose that a beam of cathode rays is incident at an angle θ on a plane of indices (hkl) of a small element of crystal. According to the Bragg formula it will be reflected provided that $2d \sin \theta = n \lambda$, where d is the spacing between parallel planes of the type (h, k, l). If L is the distance of the plate, this will give rise to a mark on the plate at a distance D/2 from the central spot where $D = 4 \theta L = 2n \lambda L/d$, assuming θ is small.

If a large number of small crystals are present arranged at random so that some are present at all angles we shall have a ring of diameter given by the above formula for every spacing d in the crystal lattice and $n = 1, 2, 3$, etc. This is, of course, the well-known Hull-Debye-Scherrer pattern for powdered crystals.

FIG. 2.

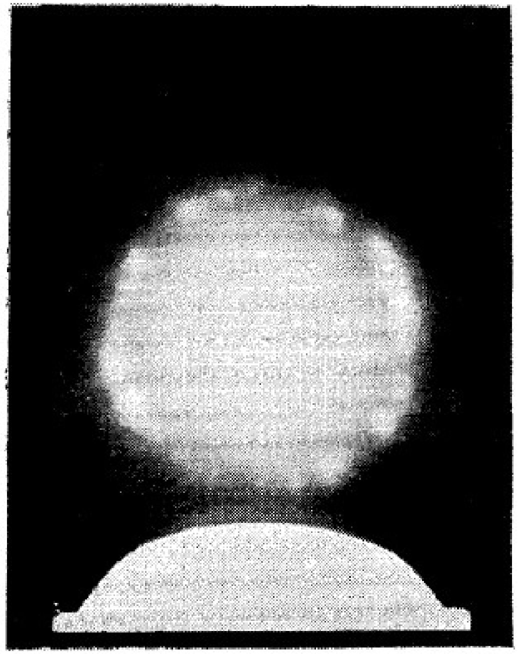


FIG. 1.—Aluminium.

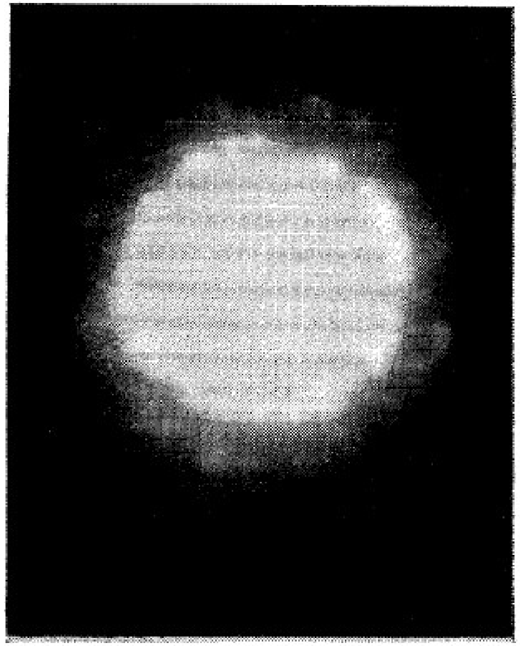


FIG. 2.—Aluminium.

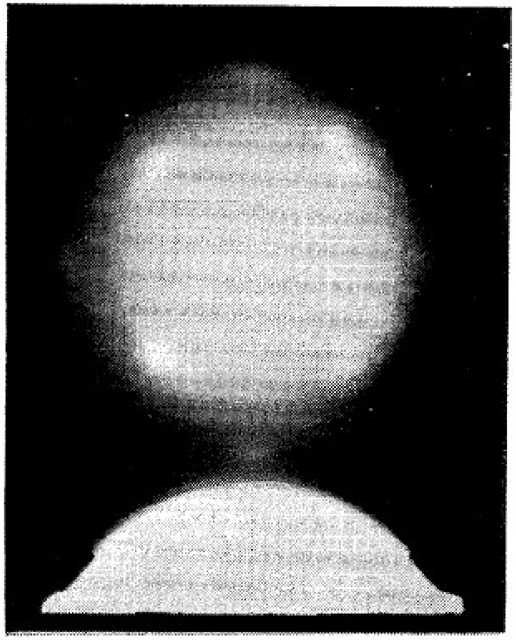


FIG. 3.—Aluminium.

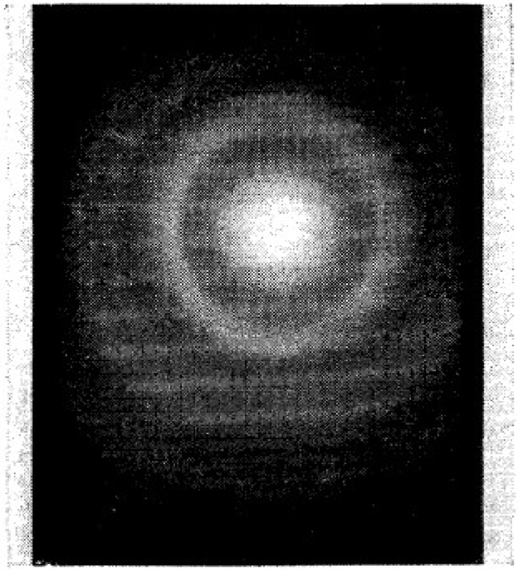


FIG. 4.—Gold.

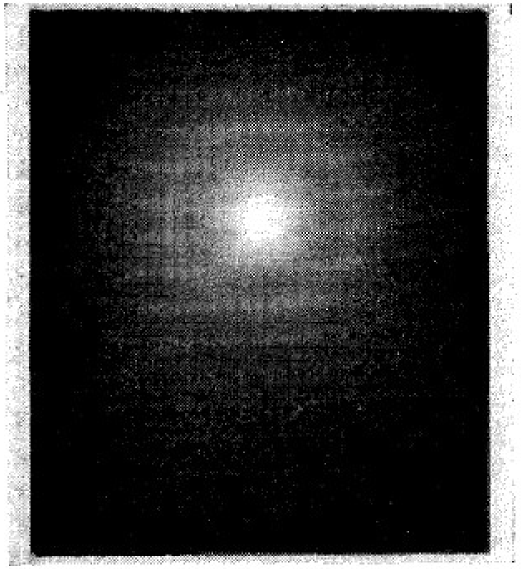


FIG. 5.—Celluloid.

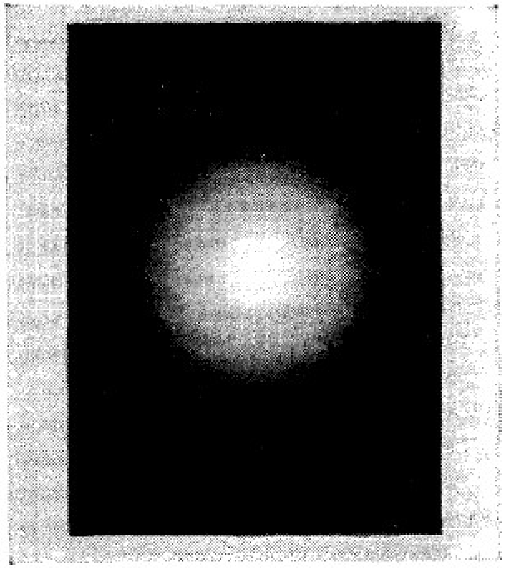
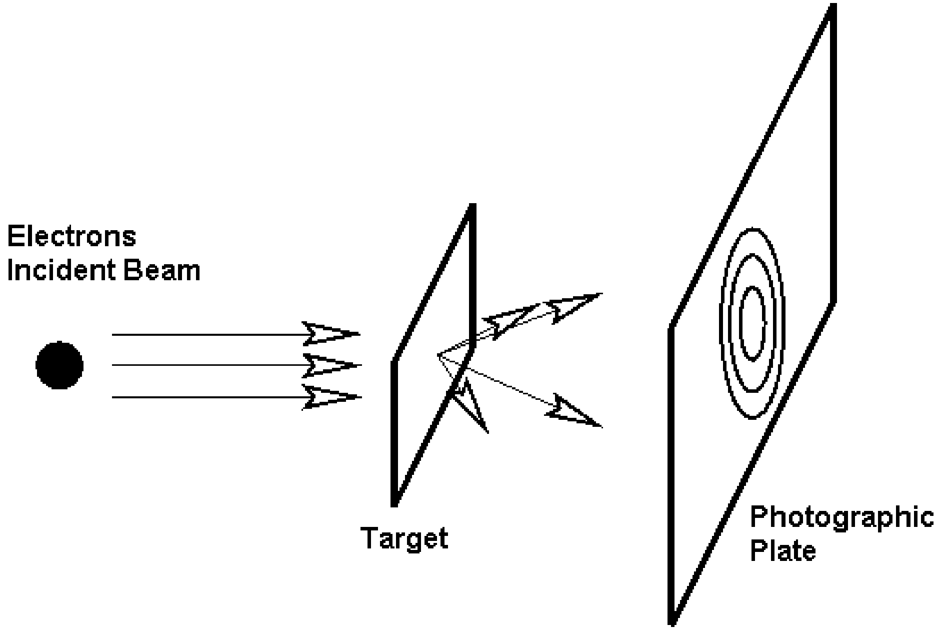


FIG. 6.—Film X.

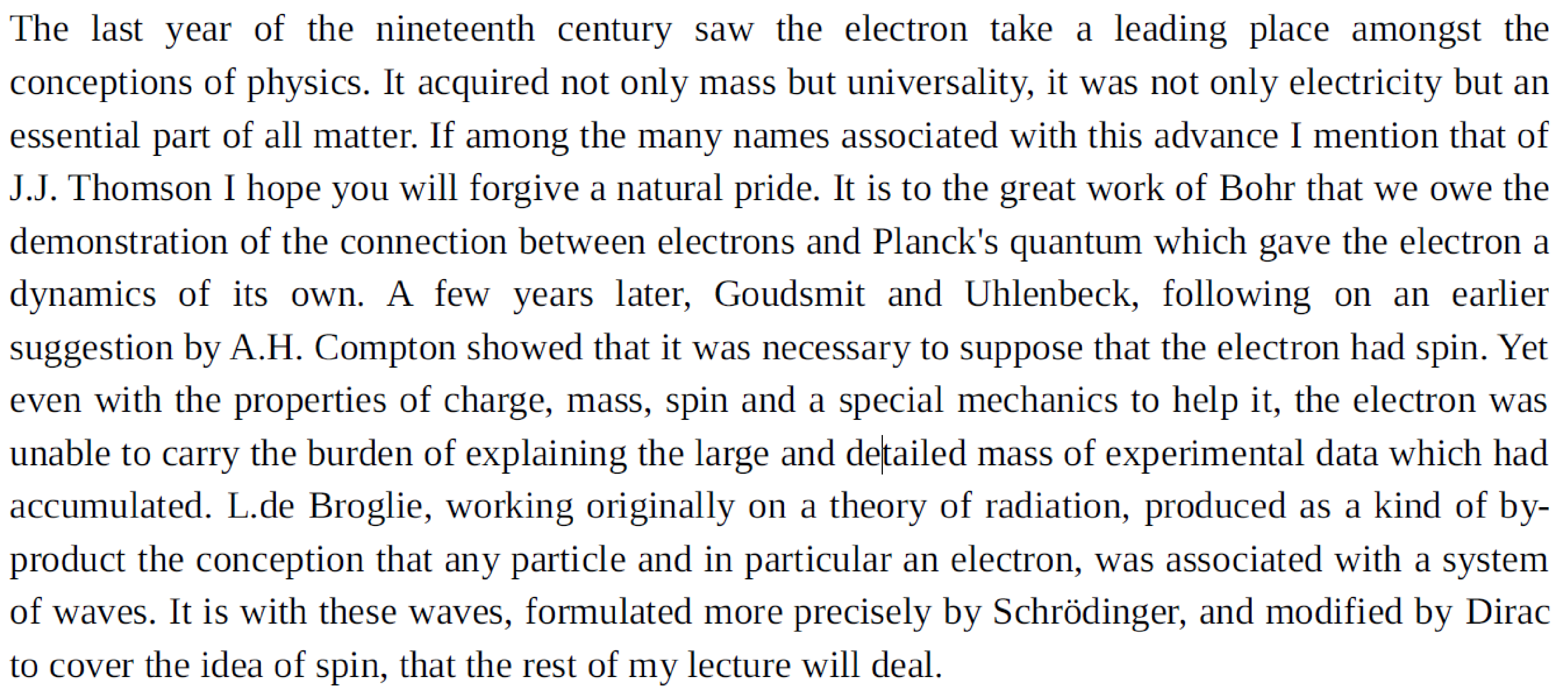
Conclusions.

The detailed agreement shown in these experiments with the de Broglie theory must, I think, be regarded as strong evidence in its favour. These means accepting the view that ordinary Newtonian mechanics (including the relativity modifications) are only a first approximation to the truth, bearing the same relation to the complete theory that geometrical optics does to the wave theory. However difficult it may seem to accept such a sweeping generalisation, it seems impossible to explain the results obtained except by the assumption of some kind of diffraction, and the numerical agreement with the wave-length given by the theory is striking. It should be emphasised that there are no adjustable constants; the agreement is direct except for a 5 per cent. error.



It is interesting to notice that
J.J. Thomson was awarded the Nobel
prize for the discovery
of the elementary particle
called electron.
His son was awarded the Nobel prize
having proved the undulatory nature
of electrons.

From G.P.Thomson's Nobel lecture



The last year of the nineteenth century saw the electron take a leading place amongst the conceptions of physics. It acquired not only mass but universality, it was not only electricity but an essential part of all matter. If among the many names associated with this advance I mention that of J.J. Thomson I hope you will forgive a natural pride. It is to the great work of Bohr that we owe the demonstration of the connection between electrons and Planck's quantum which gave the electron a dynamics of its own. A few years later, Goudsmit and Uhlenbeck, following on an earlier suggestion by A.H. Compton showed that it was necessary to suppose that the electron had spin. Yet even with the properties of charge, mass, spin and a special mechanics to help it, the electron was unable to carry the burden of explaining the large and detailed mass of experimental data which had accumulated. L.de Broglie, working originally on a theory of radiation, produced as a kind of by-product the conception that any particle and in particular an electron, was associated with a system of waves. It is with these waves, formulated more precisely by Schrödinger, and modified by Dirac to cover the idea of spin, that the rest of my lecture will deal.