## Coordinates

Coordinates are the numbers used to tell us how to get to a certain point on a grid - eg, a graph or map. The grid will have an $\mathbf{x}$ axis which runs horizontally, and a $\mathbf{y}$ axis that runs vertically.


Coordinates are written in pairs. The coordinate of $\mathbf{P}$ on the graph above is $(2,1)$. The first number in the pair is known as the $\mathbf{x}$ coordinate $(2,1)$.

The $\mathbf{x}$ coordinate tells us how many units to go across (to the left if it is a negative number, or the right if it is positive).

The $\mathbf{y}$ coordinate tells us how many units to go up if the number is positive, or down if the number is negative.

Look at these coordinates and work out which letter represents them on the graph. Write the letters down and they will form a word.
$(5,0)(-3,4)(5,-2)(-4,-1)$


The answer is 'EASY'.

Now spell the word $(0,5)(4,-3)(2,-5)(5,0)$, and check your answer at the bottom of the page.


The answer is BITE.

## Straight-line graphs

Straight-line graphs are easy to spot.

The equation of a straight-line graph can contain:

- an x term,
- a y term,
- and a number.

Although some equations for straight lines might have only two of those, like an $x$ term and a number, eg $\mathrm{x}=5$.

These are all equations of straight lines:

- $x=y$
- $x=7$
- $y=-1$
- $y=3 x+2$
- $y+x=5$
- $2 y-3 x=7$

In a typical question, you will be asked to fill in a table of values, plot them and join them up to make a straight-line graph.

Question: Complete the table of values for $\mathbf{y}=\mathbf{x}+\mathbf{3}$, and then draw the graph.

| x | -3-2-1012 ${ }^{\text {a }}$ |
| :---: | :---: |
| $y=x+3$ | 45 |

## Graph plotting

On the previous page we worked out the values of $y=x+3$. These points can be plotted on on a graph. Each pair of values in the table is an $(x, y)$ coordinate $-\operatorname{eg}(-3,0)(-2,1)(-1,2)(0,3)$ etc.

## $\mathrm{x} \quad-3-2 \mid-10123$ <br> $y=x+30123456$

Take a look at the graph $\mathrm{y}=\mathrm{x}+3$ and see how the values are plotted.


If the question does not ask you to complete a table of values first, you can still create one by making up your own values for $\mathbf{x}$. You should work out a minimum of 3 points for a straight-line graph, in case one of them is wrong.

## Finding the gradient of straight lines

Sometimes you are given a graph of a straight line and you need to find its gradient.
To find the gradient of a straight line:

- choose any two points on the line
- draw a right-angled triangle with the line as hypotenuse
- use the scale on each axis to find the triangle's:
- vertical length
- horizontal length
- work out the vertical length $\div$ horizontal length
- the result is the gradient of the line


The following graph shows the exchange rate for euros $€$ and United States dollars \$ in March 2011.

There are two marked points at $(0,0)$ and $(70,98)$.
By working out the gradient of the graph, we can find the exchange rate from euros to United States dollars.

- The vertical distance between $(0, \mathbf{0})$ and $(70,98)$ is 98.
- The horizontal distance between $(\mathbf{0}, 0)$ and $(\mathbf{7 0}, 98)$ is 70 .
- $98 \div 70=1.4$

This means that 1 euro is equal to 1.4 United States dollars.

## Curved graphs

Drawing a curved graph is similar to drawing a straight-line graph and you also have to substitute numbers into the equation.

With a curved line graph the formula will include $x^{2}$, or some other power of x .
For a curved graph, you need as many points as possible to make it accurate.

## Example

Complete the table for $\mathrm{y}=\mathrm{x}^{2}+2$.

| x | 01234567 |
| :--- | :--- |
| $\mathrm{y}=\mathrm{x}^{2}+2$ |  |

We know that $\mathrm{y}=\mathrm{x}^{2}+2$, so we need to square (multiply by itself) each x value and add 2 .
We could plot these points on a grid. Each pair of values is an (x, y) coordinate - eg, ( 0,2 ), (1, 3), (2, 6) etc.


Question: Complete the table for $\mathrm{y}=2 \mathrm{x}^{2}-10$. Then draw the graph for the equation.
Use your graph to find the value of $x$ when $y=15$
((The value of $x$ when $y=15$ should be about 3.5 . ))

## Graphs modelling real situations

Graphs are a good way to show formulae in visual form.
For example, a taxi firm uses the following graph to work out the cost of a journey.


We can find out lots of different information from this graph.

The labels on each axis are very important. The horizontal line - or x axis - shows the number of miles. The vertical line - or y axis - tells us the cost.

There are two marked points at $(0,2)$ and $(10,7)$.

Read up from 6 miles on the x axis to the straight line. When you reach the line move across horizontally until you reach the y axis. The cost is $£ 5$. This means a journey of 6 miles will cost $£ 5$.

Now try these questions.
Question: How much would a journey of 9 miles cost?
Answer: $\qquad$
Question: Jo pays $£ 7$ for a taxi journey. What distance did she travel?
Answer: $\qquad$

The graph ranges up to a distance of only 11 miles.
If we want to find out the cost of a journey longer than this, we first need to find more information from the graph.

The graph starts at 0 miles. The point $(0,2)$ tells us that 0 miles costs $£ 2$.

## $\mathfrak{£ 2}$ is the fixed charge.

We can work out the gradient by comparing the distance between two marked points on the line.
The vertical distance between $(0,2)$ and $(10,7)$ is 5 .

The horizontal distance between $(0,2)$ and $(10,7)$ is 10 .
Using the formula vertical length $\div$ horizontal length, we find:
$5 \div 10=0.5$

## The gradient of the line is $\mathbf{0 . 5}$.

This tells us that for each mile the cost is $£ 0.50$.
From this information, we can find that a journey of 18 miles would cost:

- $£ 2+18 \times £ 0.50$
- $£ 2+9$
- = £11

We can use this information to find a formula for the graph.
The formula for the cost, C , for a distance of n miles, is:
$\mathrm{C}=\mathbf{2 + 0 . 5 n}$
Using the formula we can find the cost for a mileage greater than the range of the graph.

## Graphs modelling real situations - Practise

Now practise finding information from graphs that model real situations.
John was on a cycling holiday. He drew this graph to show how far he cycled on one day.


Question: How far did John cycle that day?
Answer: $\qquad$

Question: John stopped to mend a puncture. What time was that, and how long did it take?
Answer: $\qquad$
Question: When did John travel at his fastest speed? What was that speed?
Answer: The fastest speed is the line with the biggest gradient.
The line from 10:00 to 11:30 has a greater gradient than the line from 12:00 to 14:30. This means that John's speed was greatest from 10:00 to 11:30.

The speed is the gradient of the line.
John travelled 30 km from 10:00 to 11:30. It took him 1.5 hours to cycle this distance.

Speed $=$ distance $\div$ time

- $\quad$ Speed $=30 \div 1.5$
- Speed $=20$
$30 \div 1.5=20$
We have found the speed using the same method, vertical $\div$ horizontal.
We work out the units of the answer ( $\mathrm{km} / \mathrm{h}$ ) by referring to the axes on the graph, in this case km and hours.

Therefore John's fastest speed was $20 \mathrm{~km} / \mathrm{h}$.

## Parallel and perpendicular lines

On a graph, parallel lines have the same gradient.
For example, $y=2 x+3$ and $y=2 x-4$ are parallel because they both have a gradient of 2 .

Remember that perpendicular lines will always cross at right angles.
In this diagram, the lines $y=2 x+3$ and $y=-1 / 2 x-1$ cross at right angles.


The gradients of these lines are 2 and $-1 / 2$.
The product of the gradients is $2 x-1 / 2=-1$.
You can work out whether 2 lines are perpendicular by multiplying their gradients. The product of the gradient of perpendicular lines will always be -1 .

If lines are perpendicular, $\mathrm{M}_{1} \times \mathrm{M}_{2}=-1$


Example: Find the perpendicular line to $4 y-3 x=8$ through the point $(0,2)$.


- $3 / 4 m=-1$
- $m=-4 / 3$
- Re-arrange the equation $4 y-3 x$ $=8$ in the form $y=m x+c$
- $y=3 / 4 x+2$

The gradient is $3 / 4$
Now we need to work out the gradient of the 2 nd line. Remember that when 2 lines are perpendicular the product of their gradients is -1 . Let's call the gradient of the second line $\mathbf{m}$.

In the question we are told that the line passes through the point $(0,2)$. This means that the line crosses the $y$ axis at +2 .

So the equation of the line that is perpendicular to $4 y-3 x=8$ is $y=-4 / 3 x+2$

## Finding the gradient of a line between two points

To find the gradient of a line we need to know how many it goes up, for every one across.

## Example

Find the gradient of the line joining $(1,3)$ to $(4,9)$.
As we go from $(1,3)$ to $(4,9)$ the $y$ value increases by 6 , and the $x$ value increases by 3 . So the line goes 6 up for 3 across. So this line has a gradient of $6 / 3=2$.

Use this technique to answer the following question:

Question: Line A goes through the points $(4,9)$ and $(1,3)$. Find the perpendicular line through the point $(2$, $0)$.
$\qquad$

The gradient of line A is 2 .

We know that the product of the gradients of perpendicular lines is -1 . If we call the gradient of line B m, then: $\qquad$

A straight line always has the equation $y=m x+c ; m$ is always the gradient so we know this equation is $y=-1 / 2 x+c$

Line B passes through the point $(2,0)$. To find out the value of $\mathbf{c}$ in the equation $y=-1 / 2 x+c:$ $\qquad$ The equation of line B is therefore $y=-1 /{ }_{2} x+1$


## $\mathbf{y}=\mathbf{m x}+\mathbf{c}$

We have seen graphs in the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ before.

When a graph is written in the form $\mathrm{y}=\mathrm{mx}$ $+\mathbf{c}, \mathbf{m}$ represents the gradient and $\mathbf{c}$ represents the $y$ intercept.

Take the example of the cost of a taxi ride at $£ 1+£ 3$ per mile. In this case the gradient is the cost per mile and the intercept the $£ 1$ standing charge.

This can be written as $\mathrm{y}=3 \mathrm{x}+1$

This line has a gradient of 3 and a $\mathbf{y}$ intercept of 1 .
We can use this information to plot the graph, without drawing a table.
The line cuts the $y$-axis at 1 so the y intercept is 1 .

The line has a gradient of 3 because every time it goes along one square it moves up 3 squares (every mile costs $£ 3$ ).

Question: Plot the graph of $y=-x+2$ without using a table
Answer: $m=-1$ and $c=2$, so the line cuts the $y$ axis at 2 and has a gradient of -1 .

The graph should look like this:


Remember, if the gradient is negative, the graph will slope up to the left.

## Equations

Graphs can also be drawn from an equation. These equations can show the link between two different quantities. For example if we are changing from one unit to another (for example, $£$ s to $\$ \mathrm{~s}$ ) we can draw a graph that not only shows the comparison but that we can use for a ready reckoner.


If an algebraic equation can be written in the form $y=m x+c$ then we can draw a straight line graph.

Remember that $\mathbf{m}$ stands for the gradient or slope of a line and $\mathbf{c}$ is the point at which the line cuts through the $y$ axis.

If the line cuts through the origin then the equation would become $y=m x$

## Example 1

Draw the straight line represented by the equation $y=2 x$

## Method

To draw this graph you first of all have to work out at least 3 co-ordinates Using only 2 isn't a good idea as you could have made a mistake, more than 3 and you are spending a lot of extra time calculating.

You can pick any 3 values for $\mathbf{x}$. e.g. $\mathrm{x}=2,4$ and 6 .

Then substitute these values to find the corresponding $\mathbf{y}$ value.

- when $x=2$ then $y=2 \times 2=4$
- when $x=4_{\text {then }} y=2 \times 4=8$
- when $x=6$ then $y=2 \times 6=12$

We now have the co-ordinates as $(\mathbf{2}, \mathbf{4})(\mathbf{4}, \mathbf{8})$ and $(\mathbf{6}, \mathbf{1 2})$.

You can now pick the scale, plot the points and connect them in a straight line.


Obviously x scale could be from 0 to 10 and y from 0 to 15 or 20.
Example 2: Draw the straight line represented by the equation $y=3 x+2$

Method: Again you pick 3 values for $\mathbf{x}$. e.g. $\mathrm{x}=1,4$ and 7
Now substitute to find $y$.

- $y=3 \times 1+2=5$
- $y=3 \times 4+2=14$
- $y=3 \times 7+2=23$

Co-ordinates are $(\mathbf{1}, \mathbf{5})(\mathbf{4}, \mathbf{1 4})$ and $(7, \mathbf{2 3})$.

Again you can now draw your graph in confidence that the scale is correct. Perhaps x drawn from 0 to 10 and y from 0 to 30.


A line segment is part of a line which has two end points (ie, it is not infinite). Each end of a line segment is usually labelled with letters.

## The length of a line segment

If $A$ is the point $(1,1)$ and $B$ is the point $(4,5)$, what is the length of the line segment $A B$ ?

It is not easy to picture this without drawing a sketch. Have a look at the diagram below:


A has an x-coordinate of $1 . B$ has an x-coordinate of 4 . So, to get from A to B, we move along 3 units.

A has a y-coordinate of 1 . B has a y-coordinate of 5. So, to get from A to B, we move up 4 units.

We have created a right-angled triangle. So to find the length of AB we use Pythagoras' theorem.
$\mathrm{AB}^{2}=3^{2}+4^{2} ; \mathrm{AB}^{2}=25 ; \mathrm{AB}=\sqrt{25} ; \mathbf{A B}=\mathbf{5}$

Question: If $P$ is the point $(1,5)$ and $Q$ is the point $(5,1)$, what is the length of the line segment PQ?
Answer: $\qquad$


$$
\begin{aligned}
& \mathrm{PQ}^{2}=4^{2}+4^{2} \\
& \mathrm{PQ}^{2}=32 \\
& \mathrm{PQ}=\sqrt{32} \\
& \mathbf{A B}=\mathbf{5 . 7} \text { (1 d.p. })
\end{aligned}
$$

Note: If you get a question like this, it is fine to draw a sketch or diagram. If you get really confident, you might be able to answer the question without using a diagram.

## The midpoint of a line segment



If X is the point $(1,1)$ and Y is the point $(3,5)$, what is the midpoint of the line segment XY?

Look at the diagram below:

It is clear from this diagram that the midpoint of $(1,1)$ and $(3,5)$ is $(2,3)$. In fact, the x -coordinate of M is the average of the x -coordinates of X and Y And the $y$-coordinate of M is the average of the y -coordinates of X and Y .

So: $\quad M=\left(\frac{1+3}{2} \cdot \frac{1+5}{2}\right)=(2,3)$

Question: What is the midpoint of $(2,7)$ and $(4,1)$ ?
Answer: $\qquad$

## The straight line

## Lengths, gradients and midpoints

There are several basic facts and equations connected with straight lines that you need to know by heart. Remind yourself of the straight line basics below - you'll need to use these processes in the next section.

The distance between two points, $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by the formula $\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

So the distance between $(2,3)$ and $(1,5)$ is

$$
\sqrt{(1-2)^{2}+(5-3)^{2}}=\sqrt{(-1)^{2}+(2)^{2}}=\sqrt{5}
$$

The gradient $m$ between two points $\left(x_{1}, y_{1}\right)_{\text {and }}\left(x_{2}, y_{2}\right)_{\text {is given by the formula }}$ $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ where $x_{2} \neq x_{1}$

If $x_{2}=x_{1}$ then the gradient is undefined. The gradient between $(2,3)$ and $(1,5)$ is $m=\frac{5-3}{1-2}=-2$
If a line with gradient $m$ makes an angle $a^{\circ}$ with the positive direction of the x -axis then $m=\tan a^{\circ}$

$3=\tan a^{\circ}$ so $a=71.6^{\circ}$


$$
m=\tan 120^{\circ} \text { so } m=-\sqrt{3}
$$

## Equation of a straight line

The general equation appears as $A x+B y+C=0$
However to build up an equation use $y-b=m(x-a)$ where $m$ is the gradient and $(\mathrm{a}, \mathrm{b})$ is on the line.
To identify features compare with the form $y=m x+c$ where $m$ is the gradient and $(0, \mathrm{c})$ is on the line.
Find the equation of the line with gradient 3, passing through (4, 1).
Using $y-b=m(x-a)_{\text {with } \mathrm{m}}=3,(\mathrm{a}, \mathrm{b})=(4,1)$ we get
y-1 $=3(x-4) \quad--->y-1=3 x-12 \quad--->y=3 x-11$

Find the gradient of the line with equation $2 x+5 y-6=0$.
Rearrange this in the form $y=m x+c$.
We get

$$
\begin{aligned}
2 x+5 y-6 & =0 \\
5 y & =-2 x+6 \\
y & =-\frac{2}{5} x+\frac{6}{5} \\
\text { gradient } & =-\frac{2}{5}
\end{aligned}
$$

## Co-ordinates of midpoints

To find the co-ordinates of the midpoint between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ use the midpoint formula $=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$

If $\mathrm{A}=(2,3)$ and $\mathrm{B}=(6,-4)$ then the midpoint of AB
$=\left(\frac{2+6}{2}, \frac{3+(-4)}{2}\right)$
$=\left(\frac{8}{2}, \frac{-1}{2}\right)$
$=\left(4,-\frac{1}{2}\right)$

## Parallel lines have equal gradients

For example, two lines have equations $y=m_{1} x+3_{\text {and }} y=m_{2} x-7$. If the lines are parallel then $m_{1}=m_{2}$ and if $m_{1}=m_{2}$ then the lines are parallel.

For example, show that the lines with equations $3 x-6 y-8=0$ and $2 y=x+1_{\text {are parallel. }}$.
First rearrange each equation in the form $y=m x+c$

$$
\begin{aligned}
2 y & =x+1 \\
y & =\frac{1}{2} x+\frac{1}{2}
\end{aligned}
$$

Identify the first gradient:
gradient $=\frac{1}{2}$
$3 x-6 y-8=0$

$$
\begin{aligned}
-6 y & =-3 x+8 \\
y & =\frac{-3 x}{-6}+\frac{8}{-6} \\
y & =\frac{1}{2} x-\frac{4}{3}
\end{aligned}
$$

Identify the second gradient:
gradient $=\frac{1}{2}$
Complete the proof:
Gradients are equal, hence lines are parallel.

## Equations of lines where the gradient is undefined

Equations for lines which are parallel to the y-axis cannot be determined from the above formulae. Just write the equation down!


Lines parallel to $y$-axis.
In the same way, the equations of lines which are parallel to the x -axis can also be written down.


Lines parallel to x -axis

